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Technological forecasting models and their applications in capital recovery

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Technological forecasting models

and their

applications in capital recovery

by

Kimbugwe A. Kateregga

A Dissertation Submitted to the

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I. DEFINITIONS AND KEY WORDS

This chapter presents some key words and definitions used in the body of the dissertation. They pertain mainly to the fields of life analysis and life estimation, capital recovery, technological forecasting, and statistical analysis. The term technological forecasting within life analysis has come to mean the forecasting of the process from birth to death of a product. In other areas of study, it might have a narrower meaning, e.g., the prediction of the birth of a new technology. As a result, some of the terms associated with technological forecasting might not have consistent meanings across various disciplines.

Life analysis

The statistical analysis of the information from retirement records to be used as an input into the life estimation process to determine probable lives of industrial property.

Life estimation

The use of life analysis, in conjunction with an assessment of future conditions and competent technical judgment to determine probable lives of industrial property.

Life indicators

The quantitative information about lives of equipment derived from life analysis and life estimation. For example: service life, average service life, average remaining life, maximum life.

Force of mortality

Any one of several factors ultimately leading to retirement of property. Marston, Winfrey and Hempstead [32] categorize these forces as: a) physical, e.g., wear and tear, accident, deterioration from use, deterioration from time, catastrophe, b) functional, e.g., inadequacy, technological obsolescence, c) factors unrelated to the property, e.g., termination of need, abandonment of the enterprise, requirement of public authority.

Traditional life estimation methods

These include Iowa type survivor curves, Gompertz-Makeham formulas, h-curves, computed mortality, simulated plant record, etc., all of which aggregate the forces of mortality before forecasting lives. "Traditional" is to distinguish these methods from those methods like the ones considered in this study and other contemporary studies that are trying to disaggregate the forces of mortality before forecasting lives.

Capital recovery

Fitch, Wolf, and Bissinger [13] call it depreciation as used in the context of engineering economy and quantitative management sciences. They define it as the allocation of the capital investment to accounting periods over a span of time in order to produce meaningful financial statements as the basis for rate and financial regulation.

Depreciation

The literature is full of different definitions of depreciation, with each field of study espousing a particular definition. In the area of capital recovery, the Federal Communications Commission's [8] definition reads:

> Depreciation, as applied to depreciable telephone plant, means the loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of telephone plant in the course of service from causes which are known to be in current operation, against which the company is not protected by insurance, and the effect of which can be forecast with a reasonable approach to accuracy. Among the causes to be given consideration are wear and tear, decay, action of the elements, inadequacy, obsolescence, changes in the art, changes in demand and requirements of public authorities.

Reserve deficiency

A short fall in the capital recovery process, dependent on a specific life forecast, indicating the difference between what should have been recovered and what is actually recovered at a point in time.

Technological forecasting

Bright [4] defines it as a quantified prediction of the timing and the character or degree of change of technical parameters and attributes associated with the design, production, and use of devices, materials, and processes, according to a specified system of reasoning. Jantsch [20] defines it as: the probabilistic assessment, on a relatively high confidence level, of future technology transfer. Martino [33] argues against the necessity of "a relatively high confidence level".

<u>Substitution</u>

The process as one technology replaces another in satisfying a specified need or providing a specified service. An example is the historical substitution of diesel locomotives for steam locomotives in the railroad industry.

<u>Adoption</u>

The process as society or any group of potential users utilize a new technology. An example is the adoption of radio receivers in the household. Adoption may be regarded as the substitution of having a specified technology for not having it.

<u>Growth</u>

The development and progress in the use of a technology from its introduction to its ultimate limit. This term may be generically used to refer to both substitution and adoption especially in those cases where it is not clear whether it is substitution or adoption taking place. For example, in the use of the pocket calculator, one might ask whether society is merely substituting for the slide rule or adopting the power and wider range of functional capability of the calculator.

<u>Growth model or growth curve</u>

A curve designed to indicate the general pattern of growth of a technology. Many growth curves take an S shape. Some are symmetric and others nonsymmetric.

Penetration level

The amount of growth (usually specified as a percentage of the ultimate limit) achieved by a technology at a point in time. For example, if complete substitution or adoption is expected, one may refer to the 25% penetration level when a quarter of the ultimate limit is achieved.

Life cycle

The depiction of the growth and decline of a technology.

Linear estimation

A process that first reduces a mathematical model so that it is linear in the parameters of the model before the parameters are estimated. "Linear" refers to the condition when the response variable is made up of two or more additive components of the parameters. For example, in $y_i = \alpha + \beta t_i + \varepsilon_i$, since α and β are in in separate additive components of the response variable y_i, the equation is linear and linear estimation techniques can be used for the estimation of α and β.

Nonlinear estimation

A process that does not reduce the model to a linear form before estimating the parameters but relies on a trial and error routine or a search process to arrive at parameter estimates.

Analysis of variance (ANOVA)

A statistical method that separates the effects of interest from the uncontrolled or residual variation.

One way analysis of variance

In this type of analysis, the observations are split into a number of mutually exclusive categories. The ANOVA then differentiates between those categories. For example, the investigator may use a one way analysis of variance to decide if there are differences in the fuel consumption of three types of automobile engines. The observations in this case would be grouped according to type of engine.

Blocking

When there is more than one effect to be considered, for example winter versus summer consumption of fuel in three types of engines, the observations have to be grouped not only according to type of engine but also according to season. This kind of categorizing two or more such variables of interest is called "blocking".

Two way analysis of variance

In the blocking example given above, one would use a two way analysis of variance to differentiate between the two effects of interest, namely type of engine and season.

CHAPTER II. INTRODUCTION

Depreciation is a major component in the estimation of the cost of production and service in all investor-owned businesses. For public utilities, it is even more crucial because of their capital intensive nature and the regulatory process that requires the distribution of capital costs over the service lives of assets. Life analysis and estimation is a prerequisite in depreciation allocation and capital recovery. The techniques used in life analysis and estimation have undergone numerous conceptual and technical variations over the years to evolve into the generally accepted techniques used today. Iowa type curves, Gompertz-Makeham formulas, h-curves, simulated plant records, computed mortality, etc., are all relatively well understood tools and will be referred to in this work as the traditional methods of life estimation.

In life analysis and estimation for depreciation purposes, the procedure is a rigorous study of historical characteristics and trends and their extrapolation into the future adjusted with subjective "expert opinion and judgment". Historical retirements, sometimes referred to as mortality data, are of major interest because they reflect an aggregation of all the factors - physical, functional, managerial, technological, etc., that ultimately lead to retirement.

It is commonly believed that, in the past, retirements of plant and equipment were due primarily to physical deterioration and that, of late, functional, technological and competitive factors are playing a major role in the retirement of plant. Although this belief can be justified in some cases, it has frequently led to the misconception that traditional life analysis methods, having been developed in an era of relatively slow technological advancement, fail to model the retirement patterns of today's plant and equipment. Lenz [25], for example, argues that traditional life analysis and estimation methods model the "expected lifespans for equipment which is presumed to suffer wearout or other types of physical deterioration". Hawkins, Paulson and Wallace [18] also state that "... mortality analysis models ... are driven essentially by physical deterioration or wearing out of equipment". These views are contrary to Marston, Winfrey and Hempstead [32] who, in addition to the other forces of mortality like wear and tear and accident also list as forces modeled by traditional methods:

> Obsolescence, another characteristic of functional undesirability [which] is usually brought on by the invention and development of improved devices .... Style changes and supersession [which] cause obsolescence when the same service can be rendered with greater economic efficiency by a different kind of structure or equipment. An example would be the substitution of electric motors or internal combustion engines for steam generator engines in plants where the former would be more economical.

Traditional life analysis techniques would just work on the retirement data without specifically disaggregating the factors. Thus, if technological causes were predominant, they would be assumed to be fully reflected in the retirement data and appropriately accounted for.

Fitch and Wolf [12] identified the need to examine individual forces of mortality and conceptualized on how those forces could be combined to give better life forecasts. Wolf [51] in particular supported Ocker [38] in the belief that obsolescence and technological improvement is a single force of mortality dominating all others in the telecommunications industry and should thus be studied separately. Wolf identified three steps necessary to obtain a life forecast when a particular force is disaggregated. For technological obsolescence, he enumerates:

- the estimation of the effect of all forces except technological obsolescence,
- the forecasting of the future rate of obsolescence, and
- the combination of these forces of mortality to yield a service life forecast.

He points out that the most critical of the three steps is forecasting future life cycles (as a surrogate to obsolescence).

Dandekar [6] argues that the concept of the retirement rate being a function of age, as used in some traditional life analysis techniques, might have to be augmented to a more universal concept that not only relates the retirement rate to age but also to chronological time. This new dimension of life analysis will, more appropriately, account for the technological and obsolescence factors leading to retirement.

In their preface to "The Estimation of Depreciation", Fitch, Wolf, and Bissinger [13] state:

> The effect of advances in technology, technological forecasting, life cycle costing and life cycle depreciation are current topics related to depreciation which need to be incorporated in [life estimation] studies.

This study has taken the stand that traditional life analysis techniques have to be complemented with technological forecasting techniques to give better life estimates. Neither of the methods can claim to fully reflect all the relevant information necessary for life estimation, and neither can brand itself as being more futuristic or historical than the other. Each method relies on historical data to predict the future although each method does so from a different perspective. For example, the traditional methods focus on intra-account information without disaggregating it into subaccounts, while a method like substitution analysis focuses on information across accounts or across subaccounts without consideration to intra-account information. Additionally, traditional methods, by mixing age and time relationships, confound the forecasting problem; and substitution analysis, by ignoring the age relationship of retirements and failing to address the nature of addition and retirement patterns, sidesteps a part of reality. By exploiting the strength of each method, and combining the resulting forecasts into a singular prediction, better life estimations should be obtained.

In a technology driven environment, it seems imperative, then, to complement traditional life analysis with technological forecasting techniques. Resort to technological forecasting is based on the premise that, if one can forecast not only the onset but also the pattern of development of particular technologies, one can estimate better the lives of affected equipment.

CHAPTER III. THE NEED FOR TECHNOLOGICAL FORECASTING: A CASE STUDY

The telecommunications industry has had one of the fastest rates of technology development in the recent past. The very vocabularly in the field - microprocessors with high density electronic memories and megabit DRAMS, fiber optics and optical (light wave) technology, super semiconductors with gallium arsenide (GaAs) compositions, artificial intelligence, electronic voice recognition, video conferencing, digital and packet switching, local area networks (LANs), integrated services digital networks (ISDN) - attests to an extremely fast rate of technical development.

The telecommunications industry, especially the regulated telephone companies, maintains that technology advancements coupled with competition are having a drastic effect on the lives of its equipment and its capital recovery process and are a main driver of the embedded reserve deficiency. The telephone industry is of a highly capital intensive nature. Book value of U.S telephone companies' physical assets exceeds $190 billion (Forbes [15], Grabhorn [17]). Depreciation alone accounts for about 30 percent of the industry's revenue requirement and in some companies, e.g., Illinois Bell, depreciation is the single largest cost. [Letter from T. L. Cox (Vice President, Finance, Illinois Bell, Chicago, IL) to W. J. Tricarico (Secretary, FCC), Re: 1984 represcription of depreciation rates for Illinois Bell, July 20, 1984.]

Regulated industries determine their annual allowed revenues from the revenue requirement equation:

$$RR = OE + T + D + ROR(RB)$$

where RR is the revenue requirement, OE is operating expense less depreciation, T is taxes, D is depreciation, ROR is the allowed rate of return and RB is the rate base so that ROR(RB) (ROR multiplied by RB) is the allowed return.

A utility can thus either claim depreciation or not claim part of it and keep the unclaimed portion in the rate base and continue to earn a return on it. In the simple case of a single unit of equipment, (for group properties the analysis gets more complex although the fundamental concept is similar), that portion of the rate base claimed as depreciation is credited to a depreciation reserve where it stays until the equipment is retired and then it goes off the books altogether. At retirement, total credits into the reserve should equal the equipment's first cost less net salvage. A reserve deficiency is created if the accumulated credits are less than what should be in the reserve at that particular point in time. The deficiency is due to underaccrual in the past on existing equipment, given a specific life forecast, and/or to retirement in the past of equipment that was not fully depreciated.

The relationship between the rate base and the depreciation expense means that the present reserve deficiency in the telephone industry is still part of the rate base and the regulated industry can continue to earn a return on the unrecovered portion of it.

Before their divestiture from AT&T in January 1984, the Bell operating companies (BOCs) operated under the monopolistic umbrella of AT&T. The divestiture, however, not only put a stop to the monopolistic control AT&T had over the telephone industry but also created more concern over the reserve deficit in the BOCs, now on their own and faced with a relatively more competitive market not playing by the rules of regulation. But even before divestiture, expanding competition and the need for competitive pricing in various business markets had already started impacting the telephone industry. Just before divestiture, AT&T estimated its depreciation reserve deficiency at over $25 billion and growing at a rate of $2 billion a year (Forbes [15]).

It is difficult to pinpoint any one factor that was individually responsible for this state of affairs. Was it life estimation, competition, depreciation methods, or politics? The capital recovery manager of Ameritech Services once pointed out that:

> Claims and counter claims have been hurled back and forth between regulators and telephone managers over responsibility for the reserve deficiency. There seems to be no continuing debate over its existence. There are differing "estimates" of its exact size, but no one is denying it's here or that it's big. Both sides have "proof" of the others fallibility. The FCC staff has AT&T statements from the early 1970's denying its existence (the "you didn't tell us" argument) while management has documented records of the reverse argument ("oh yes we did") dating back to the 1950's (Nousaine [37]).

That some blame falls on life estimation is probably true, as can be concluded from Chapter I. But even more pertinent to the deficiency problem has been the slow acceptance of such refined methods of depreciation allocation as vintage group and equal life group procedures and remaining life techniques which tend to reduce the burden of life estimation.

The concept of "equal life group" has been around at least since Winfrey [50] published "Depreciation of Group Properties" in 1942 wherein he refers to it as "the unit summation procedure". But it was not until 1980 in the FCC docket 20188 [9] that the FCC authorized the use of the equal life group and, even then, limited it to new plant and established a phase-in of accounts. In the same docket, the FCC also instituted the remaining life method which has been around at least since the publication of Marston, Winfrey, and Hempstead [32] in 1953.

Political forces must also have fuelled the reserve deficiency as evidenced for example in docket 20188, p. 16:

> The seeming attraction of stretching out lives to hold down depreciation expense may impose longer-term costs on our society that far outweigh the short-term advantages.

The reluctance and the difficulty of changing the forecast life from one primarily determined by history to one placing more emphasis on future conditions did and still does interfere with the timely recovery of capital.

In the past, net salvage realized used to be substantial thus reducing the impact of poor estimates of lives on the reserve. The pace of technology, however, has almost eliminated the market for reusable products and has introduced substantial negative salvage which impacts the reserve adversely.

The condoning of the existence of one overall book reserve instead of reserves by account, subaccount or even by vintage could in itself have been a contributing factor to the reserve deficiency in the telephone industry.

In the competitive environment BOCs find themselves, capital recovery is of major importance. More deregulation or complete deregulation of the telephone industry is a possibility. Depreciation then would be a cost of doing business only claimable through market prices. By overdepreciating, BOCs would price themselves out of business and underdepreciating, although possibly lucrative in the short run in the form of lower prices and higher demand, would create higher prices in the future which the market could not support. The present deficiency would have to be written off the books, resulting in market repercussions whose effect is very hard to predict. The BOCs are therefore trying, while in the regulated environment, to recover as much of the deficit as they can as quickly as possible, within the constraints of pricing for an increasingly competitive marketplace.

Recovery of the deficit, however, is a dilemma of gigantic proportions. Underdepreciation in the past means that the present rate base is inflated and that telephone assets are overvalued. A competitive market cannot allow a return on overvalued assets. Trying to recover the deficiency through higher depreciation will dictate higher rates which will most certainly force big customers to install inhouse systems - the classic by-pass phenomenon. Loss of revenue through customer loss will result in higher rates for the remaining customers, setting up a customer loss vicious circle. To retain customers, the BOCs will have to reduce their real costs of operation through equipment upgrading and technological innovation to be able to compete with other carriers; and depreciation would be a desirable source of the necessary funds.

It is clear from the foregoing exposition that improved life estimation methods, especially those methods that specifically account for technological obsolescence and competitive factors, will help in reducing the present insidious depreciation reserve deficit provided those life estimations are reflected in the rates. Otherwise only partial recovery will be accomplished. As the FCC [9] states in docket 20188, p. 26:

> If the currently estimated short lives had been known all along, the past depreciation rates would have been higher ... and current reserves would be higher. Absent a reversal of the current trends and without corrective action, the amount of difference due to errors of life estimate will continue to grow.

And in docket 83-587 the FCC [10] again states:

> It is in the best interest of both the company and its ratepayers to eliminate these reserve imbalances as quickly as possible without imposing a material impact on total depreciation expenses in any one year.

Additionally future life estimations do certainly have to incorporate technological forecasting to be reliable and realistic, both in the telephone industry and in all those industries affected by technological improvements. This will certainly reduce the chances of the recurrence of such astronomical deficits as the telephone industry is facing today.

CHAPTER IV. OBJECTIVES OF STUDY

The major objectives of the study were:

1. To study several technological growth models and to find out if any particular model was or a group of models were dominantly superior to other models as forecasters of technological growth at different penetration levels.
2. To find out if nonlinear estimation improves the forecasting ability of the models at the different levels of penetration. Meade [34] argued that the use of untransformed data (nonlinear estimation) ensures that the most recent observations are given most weight, which tends to produce better forecasts, and that logarithmic transformations tend to place greatest emphasis on the early part of the curve and produce poorer forecasts.
3. It is generally accepted that fitting ability is not an indication of how well a model forecasts. It was therefore of interest to check this supposition for the different technological growth models and for the different levels of penetration.
4. If indeed technological forecasting and in particular substitution analysis is necessary in life estimation, especially in those industries faced with fast-paced technology, one objective was to recommend a technique to incorporate one or more appropriate models in the traditional

life estimation framework in order to improve the quality of life indications.

CHAPTER V. TECHNOLOGICAL FORECASTING AND GROWTH MODELS

Technological Forecasting in Perspective

Technological forecasting is defined in Jantsch [20] as the probabilistic assessment, on a relatively high confidence level, of future technology transfer. He differentiates between exploratory technological forecasting which starts from today's assured basis of knowledge and is oriented towards the future, and normative technological forecasting which first assesses future goals, needs, desires, missions, etc., and works backward to the present. One technique of exploratory technological forecasting is the extrapolation of time series after the formulation of simple analytical models. The extrapolation is based on an empirical belief that historical trends will be maintained at least in the foreseeable future ("deterministic techniques") or that they will undergo estimable gradual changes ("symptomatic techniques").

A group of deterministic exploratory models called growth models attempts to predict the behavior of maturing technologies. Many of these growth models assume that a technology will progress along an S shape pattern of growth. For all practical purposes, an adoption model is not different from a substitution model although, conceptually, the two underlying processes are different. Substitution may be defined as the process when one technology replaces another providing the same service to a potential market. Adoption on the other hand may be defined as the development of the market for a new technology providing a specific service. Since the models used for both concepts are similar, no attempt is made in this exposition to distinguish between them and they will often be referred to as growth processes and their models as growth models.

The S-shape Pattern of Growth

The S-shape pattern of growth can be described as slow initial growth followed by accelerated growth in the mid-section of the curve and decelerating growth as the ultimate equilibrium is almost achieved. A symmetric and a nonsymmetric S curve are shown in Fig. 1.

Many biological growth situations exhibit the S shape growth pattern. Pearl [39] was one of the earlier observers of this phenomenon and he formulated it into what is generally known as the Pearl-Reed or logistic curve. The logistic is a symmetric S shaped growth curve whose equation is:

$$y = \frac{k}{1 + e^{a+bt}}$$

where y is the penetration level achieved at time t, $b<0$ and a are constants, and k is the upper limit that can be achieved by y.

Technologists have observed S shape patterns in technological growth situations too. Lenz [26] is one of the pioneers who linked the biological to the technological and arrived at the same formulations.

Use of the S-curve for predictive purposes presupposes that the process will indeed grow. Like biological growth failures, technology is replete with cases of innovations or adoptions that aborted due to social, political, economic and/or other pressures. If the process does

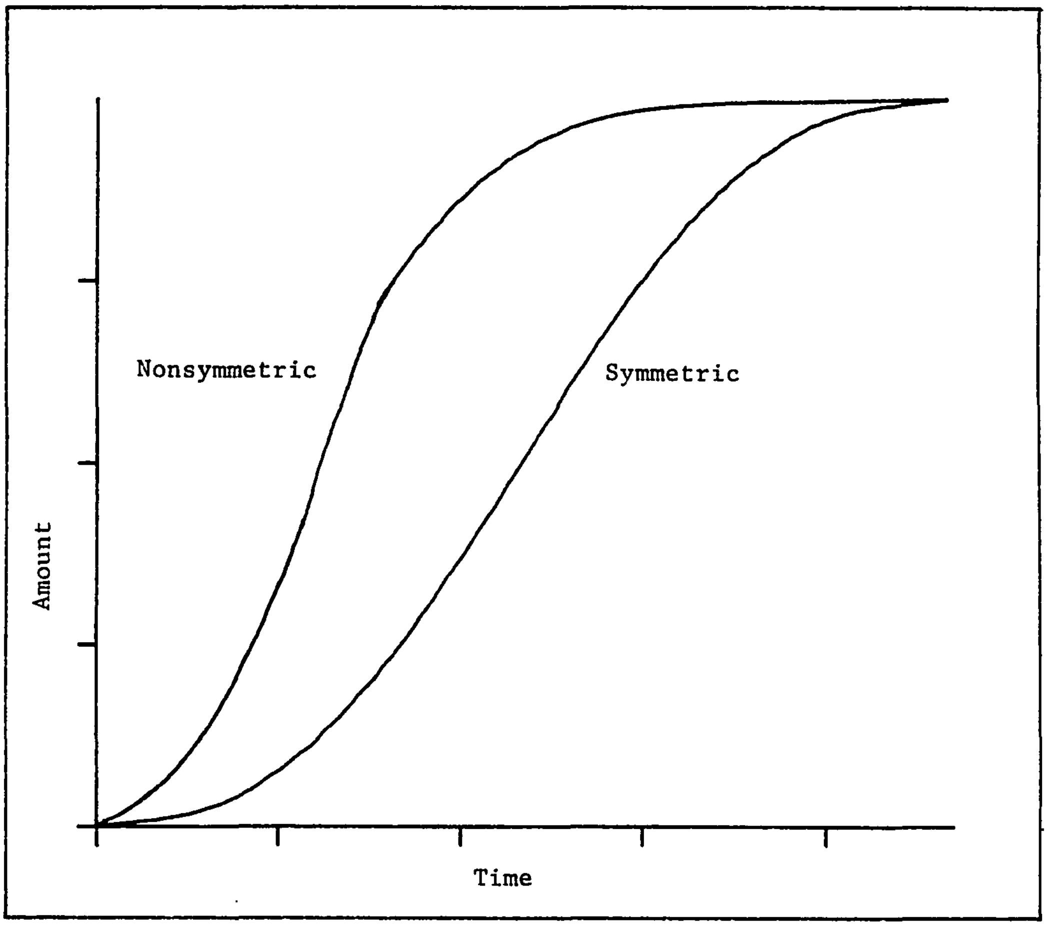

Figure 1. A symmetric and a nonsymmetric S curve

grow, however, why the S shape?

Many researchers have addressed this phenomenon and the following ideas are mainly derived from the work of Stern, Ayres, and Shapanka [47] and Lakhani [24].

Empirical reasons for the S shape

Slow initial growth During this stage, slow initial growth is experienced because the new product has to prove its superiority over existing technologies. It has to overcome the ignorance and/or resistance of consumers and the information gap between producers and consumers. Performance bugs in the earlier models have to be eliminated. Contract arrangements for the old technology are still in force and cannot be violated; there are production diseconomies due to small scale and problems of financing, developing, installing, and learning. Consumers postpone acceptance in anticipation of changes in quality and price and there is a low elasticity of supply of the new technology.

Lakhani [24] and Mansfield [30] discuss the effect of the age distribution of existing capital stock on the rate of growth during this stage. Since new processes usually require new capital equipment, firms with relatively old equipment would be prompt in accepting the new technology. The speed of take off of the new technology will then be determined by that age distribution. Mansfield, studying the effect of the age distribution relationship in the railroad industry, found that the older the steam locomotives of the firm, the faster was the rate at which that firm adopted the diesel locomotive. From a slightly different viewpoint, Salter [42] argues that firms with older technologies might continue to produce because their capital costs are considered sunk costs and their higher operating costs equal the operating plus capital costs of the firms with the new technologies.

Rapid, explosive or exponential growth In this stage, most of the bottlenecks in the initial stage have been overcome. The product has become accepted and the production processes improved. Economies of scale have set in with consequential reduction in prices; new contracts have been made and the learning process ended. The quality of the process has surpassed that of the older technology and the information gap between producers and consumers has been bridged. The bandwagon effect has begun.

Levelling off toward the equilibrium Biologically or technologically, no process can grow exponentially as described in the previous paragraph, without constraints on the growth almost automatically setting in at some stage. Technological and social economic factors will ultimately start limiting the growth of any technological process. In this latter stage of development, the product has essentially exploited its scale economies, it has matured and is no longer changing rapidly. But probably the most critical limitation is the virtual saturation of the available market. Finally, a new product could be introduced as a substitute at this time, not only forcing the growth to stop, but also initiating a decline phase.

Scientific reasons for the S shape

Lakhani [24] goes beyond the empirical and alternatively approaches these observations from an economic viewpoint. By grouping them into demand and supply factors, he argues that the growth curve might be construed as the growth of demand or supply whichever is smaller at a particular time. In the initial stage supply and demand are both restricted. In the expansion stage, increased demand and supply are driven by synergy until the equilibrium stage where demand levels off and forces supply to terminate.

Peterka [41] demonstrates that under constant productivity differentials, competing industries win and lose the market following logistic (S-shaped?) paths. Fleck [14] regarded market penetration as a diffusion process in which the buyer is a scattering element in a Markov chain ultimately leading to a logistic equation.

Marchetti [31] attempts to "reduce the empirically efficient logistic relationship to more basic and already accepted scientific axioms". He assumes that society is a learning system and if so, it is basically a random search with filters and therefore, being a random search, has to be characterized by logistic-type functions as demonstrated by Goel et al. [16] and Bush and Mosteller [5]. Marchetti studied:

- the growth process of learning of a language
- a group of people interconnected by information links and working on a common physical goal,
- a group of people again interconnected by information links and working on a conceptual goal,
- large industries capillary interconnected to many strata of society - technical, economic, financial, and political - and drawing stimuli and constraints from them,
- humanity as a whole and its behavior with respect to the use of primary energy sources during the last century.

He demonstrated that the logistic response observed in all these cases is due to the underlying fact that society is a learning system and the learning process should inevitably, scientifically, lead to logistic-type response functions.

CHAPTER VI. THE MODELS STUDIED

To satisfy objectives 1, 2, and 3 of this study, linear and nonlinear models were investigated. A linear estimation technique, in this study, is defined as the transformation of the S curve data into a linear form before the parameters of the model are estimated. Since many growth curves have formulations with exponential functions, logarithmic transformations are necessary to linearize them. Nonlinear estimation, on the other hand, derives the desired parameters through a trial and error process without relying on linearizing transformations.

Six models were selected for analysis. The very subjective criteria for selection included: track record, popularity, simplicity and potential. The mathematical formulations for each of these models are given in Appendix B.

Preeminent among technological forecasting models is the logistic curve developed by Pearl [39] and originally used in biological growth situations. The logistic was introduced in Chapter IV. Assuming complete substitution or adoption, a linear transformation of the logistic leads to a model popularly known as Fisher-Pry. It was studied and applied to a number of substitution cases by Fisher and Pry [11] of General Electric in the early 1970s. It is generally used in the form:

$$\ln \frac{y}{1-y} = b(t-t')$$

where y is the penetration level achieved at time t, b and t' are the parameters of the model, and ln is the natural logarithmic function. Fisher-Pry was selected for study because of the attention it has engendered in the telecommunications industry and its relative simplicity of formulation.

The Gompertz growth curve, also used extensively, is of the form:

$$y = Le^{-Ge^{-kt}}$$

where y is the penetration level achieved at time t, L is the upper limit, and G and k are the parameters of the model. Lakhani [24] used it to fit the technological development of processes in the petroleum industry and it is also discussed in Luker [27]. The Gompertz was selected because it is one of the oldest growth models. A derivation for the Gompertz is given in Appendix B.

The extended logistic was proposed by Mahajan et al. [28] to correct a weakness in the Bass [2] model. They called their model "the generalized logistic" but Meade [34] renamed it the extended logistic to clear up the confusion of names with an earlier "generalized logistic" proposed by Nelder [36]. The Bass model reduces to the logistic although it starts from different assumptions. The extended logistic is similar to the logistic except for the assumption of an existing level of penetration at the earliest observation time. Of the six models studied, the extended logistic could not be linearized and was thus studied only as a nonlinear model. It has the mathematical formulation (see also Appendix B):

$$y(t) = \frac{m-pZ(t)}{1+(\frac{q}{m})Z(t)}$$

where

$$Z(t) = \frac{(m-a)e^{-(p+q)t}}{p + \frac{qa}{m}}$$

y is the penetration level achieved at time t, m is the upper limit, a is an an existing level of penetration at the earliest observation time, and p and q are the parameters of the model. The extended logistic was selected because it is one of the newer models in the literature and it has been used extensively to model market behavior.

The three models - Fisher-Pry, the Gompertz and the extended logistic - are empirical behavioral models. Many other such models are discussed in the literature and Hurter et al. [19] and Meade [34] provide an extensive review of them.

A rich source of growth models is statistical and probability theory simply because many cumulative distribution functions exhibit the ubiquitous S shape. The Normal cumulative curve was selected from these for its wide applications and popularity in other areas. Stapleton [46] refers to an application of the Normal in [23] to a growth situation and uses it to fit the synthetic for natural fiber substitution. It is of the form:

$$y(t) = \int_{-\infty}^{t} (2\pi\sigma^2)^{-\frac{1}{2}} \exp(-\frac{(t-\mu)^2}{2\sigma^2})\ dt$$

where y is the penetration level achieved at time t, and σ and μ are parameters of the model. The Weibull was selected for its powerful fitting capability. It has the mathematical formulation:

$$y = 1 - e^{-\left(\frac{t-\mu}{\eta}\right)^{\beta}}$$

where y is the penetration level achieved at time t, and β, η, and μ are the parameters of the model. Kateregga [21], [22] compared it to the Iowa curves and derived capital recovery factors for various combinations of its parameter. Sharif and Islam [43] demonstrated its use in a technological growth situation.

The Lognormal is another statistical formulation that has been applied in econometric analysis, biological response situations and life analysis. Aitchison and Brown [1] have compared it to the logistic in growth situations and found the results to be "not very different". It can be expressed as:

$$y(t) = \int_{0}^{t} (2\pi\sigma^2 t^2)^{-\frac{1}{2}} \exp(-(\log(t-\tau)-\mu)^2/2\sigma^2)\ dt$$

where y is the penetration level achieved at time t, and μ, σ, and τ are the parameters of the model.

Fisher Pry, the Gompertz, the Weibull, the Normal and the lognormal were used both in their linear and nonlinear forms. The linear and nonlinear forms of the models are discussed in Appendix B.

CHAPTER VII. THE DATA ANALYZED AND THE STATISTICAL ANALYSIS

The Data

Twenty-two historical growth cases from various industries were used in the study. The main requisite for inclusion in the set was that any case have a few points (at least two) before the five percent penetration level, an increasing number of points through ten percent, twenty-five percent, fifty percent and several points beyond the seventy-five per cent penetration level. This was because the analysis was designed to check for forecasting ability at each and every one of those levels.

In all cases except two, it was assumed (where it was not conspicuously apparent) that penetration would go to a hundred percent. In the cases of "U.S.A. households with radio" and "U.S.A. households with TV" the adoption seemed to have leveled off at about ninety-nine percent for radio and ninety-seven percent for TV. Those levels were then assumed to be the upper limits and the data values were adjusted so that each value was taken as a percentage of its respective upper limit. This was done for these two cases more to illustrate a method than to achieve better forecasts. Normally however, when the estimated upper limit is well below unity, the investigator might have to resort to this method. A list of the cases and their sources is given in Appendix A.

The data used in the study for the telephone industry came from ten companies and pertained to the substitution of stored program control (SPC) for electromechanical switching in central offices. Since by 1985 only one of these companies had achieved seventy-five percent penetration, the analysis was done only up to the fifty percent penetration level. The main interest in this data set was to analyze an ongoing substitution of paramount interest to the telephone industry in the light of the information obtained from the analysis of the historical cases and to be able to suggest what models to use for this substitution.

Fitting and Forecast Errors and the ANOVA

The SAS (Statistical Analysis System) package of computer programs, version 5, was used for both linear and nonlinear estimation and forecasting and for the analysis of variance (ANOVA). The Marquardt subroutine was used for nonlinear estimation and forecasting. A forecast error was defined as:

$$e_t = y_t - \hat{y}_t$$

where e_t is the forecast error at time t, y_t is the actual penetration achieved at time t and $\hat{y}_t$ is the forecast penetration at time t. A fitting error was defined analogously. This approach is not unique and it has been used by other investigators such as Eilon et al. [7] and Nagar [35]. For example, using model A in its linear form at the 10 percent level to forecast case 5, all points for case 5 up to and including the ten percent point, if it was one of the points, would be used at the estimation stage. All points beyond the 10 percent level would be forecast and the sum of the squared forecast errors obtained.

That sum would then be divided by the number of points forecast to obtain the average squared forecast error. The average squared forecast error would then be used for model A and case 5 at the ten percent level in the ANOVA, i.e.:

$$XA5 = \frac{\sum_{t=1}^{N} (e_t)^2}{N} \bullet 1000$$

where XA5 is the average of the squared forecast errors for model A for case 5, N is the number of points forecast and multiplying by 1000 is only to avoid working with very small numbers. Thus, for the five linear models and twenty two cases at each level, an ANOVA table in the form of Figure 2 would be used.

Case	1	2 -	---	22
Model A	XA1	XA2 -	---	XA22
B	XB1	XB2 -	---	XB22
|	|	|	|	|
|	|	|	|	|
E	XE1	XE2 -	---	XE22

Figure 2. Typical table to be analyzed by ANOVA

Tables of such errors, on a case by case basis, at each level of fitting or forecasting are given in Appendix C.

A two way analysis of variance was then performed. Blocking (see definitions) across cases was considered necessary because of the suspicion of case by case variation that would not necessarily be accounted for by a one-way analysis alone. The implication for blocking in this case is that if a particular case was difficult to predict because of the nature of its data, and all models experienced this difficulty, the error due to this fact would not count against any model. The F test, at the 95% confidence level, was used in the ANOVA to compare the models A-E. When there is not sufficient evidence to discriminate among the models, the calculated F should have a value around 1, and it should become large when the models differ substantially. For example, in Table 1 of Chapter VIII, at the 75% level for the linear models, the expected F with 95% confidence is 2.5 while the calculated is 6.95, indicating substantial differences among the models. Additionally, the probability of a higher value of F, P>F, is tabulated for all comparisons. It shows the probability of mistakenly labelling the models different when in fact they are not.

Several assumptions are incorporated in the analysis of variance and serious departures from those assumptions could render the conclusions derived from the F test void. The most critical of those departures are listed by Snedecor and Cochran [45] as: lack of independence of errors, nonadditivity, heterogeneity of variances and nonnormality. Underwood et al. [48] point out however that:

> The practical usefulness of the analysis of variance procedure may be nearly as great when one or two of these assumptions are not fulfilled as when all are satisfied. If one of these assumptions appears not to be met, the experimenter may prefer to perform the analysis of variance and interpret it conservatively (e.g. require that his/her F values reach the tabled values for 99% for significance when he/she would otherwise have required only 95% level.)

Because of the small number cases involved in this type of analysis it is difficult to decide whether the assumptions of normality and equal variance have been met. Pearson [40] and Box and Anderson [3] attest to the fact that the F test for differences between means is robust to departures from normality. Moreover some of the tests on the assumptions are very sensitive to other violations of the assumptions. For example, Bartlett's test for homogeneity of variance is very sensitive to nonnormality. The random assignement of the cases to the models leaves little doubt as to the independence assumption satisfaction. For these reasons, no attempt was made to check if the assumptions were satisfied.

All statistical significance tests were performed at the 95% confidence level. Whenever the F test was significant, the Least Significant Difference (LSD) (see, for example, Wetherill [49]) was calculated to determine which models were significantly different. The LSD is calculated as:

$$LSD = s\sqrt{\frac{2}{n}} \bullet t_{0.5\alpha, c(n-1)}$$

where s is the root mean square error, n is the number of cases per model, α is the significance level and c(n-1) are degrees of freedom of the error sum of squares. The LSD shows the magnitude of the difference between any two models that would be necessary to distinguish them as different at a specific confidence level. The 95% level of confidence was used. In all cases, the models were then ranked according to their means.

Estimating the Threshold Parameter for 3-Parameter Models

The Weibull and the lognormal were used as three parameter models. To estimate the threshold parameter for the Weibull (μ) and the lognormal (τ), (see model formulations in Chapter VI and Appendix B.) a linearizing routine was used. Using μ as an example (but the discussion holds for τ as well), assume that the right value of the threshold parameter being sought is in fact μ. If one uses the value μ-2 and plots the resulting points in order to linearize them, one obtains the concave curve (1) as shown in Fig. 3 an indication that μ-2 is not the right estimate of μ. By using μ+2, one would get the convex curve (2) again indicating that μ+2 is not optimal. Since curve (1) and curve (2) are opposite in curvature, the optimal value of μ must lie somewhere between μ-2 and μ+2. By searching out for other values, say μ+1 and μ-1, one approaches the optimal value μ, where the points should plot as a straight line. Mann, Schafer and Singpurwala [29] and Aitchison and Brown [1] discuss this trial and error method of estimating the threshold parameter in more detail.

In practice, using a computer fitting routine for example, one would check for the R^2 (coefficient of determination) of curves (1), (2), (3), and (4) and pick that value of μ that gives the highest value of R^2, assuming all possible, practical values have been bracketed in the process.

In this study the initial estimate of μ was taken as the year just prior to the first observation; the next estimate as two years before the first observation and so on. There were cases when the estimation

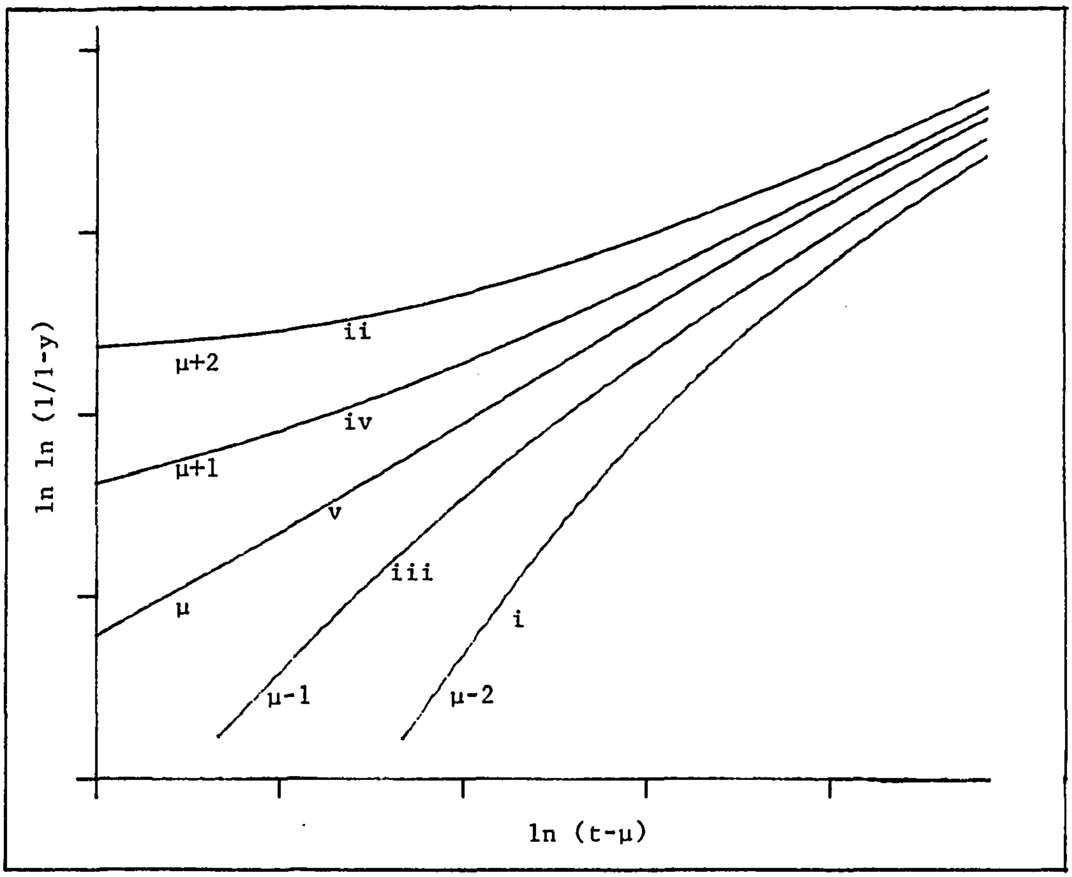

Figure 3. Graphical estimation of threshold parameters

process did not converge fast enough. In those cases, when an estimate of μ resulted in an increase in R^2 of only 0.001 or less, the estimation process was terminated and the estimate just prior to the last was used as the estimate of μ. This same estimate was used for both the linear and nonlinear estimations of the model. Although the nonlinear routine could automatically estimate the threshold parameter, the results were not deemed worth the extra computer time and expense.

CHAPTER VIII. RESULTS AND DISCUSSION

The Multi-industry Results and Discussion

The analysis of variance of the fitting ability of the models for the twenty-two multi-industry cases is shown in Table 1 where the linear comparison and nonlinear comparison across models are given. The bracketed number after the name of the model is the mean for the twenty-two cases of the average squared error as described in equation (1).

Table 2 shows the linear versus the nonlinear fitting ability of each model, at each level, ranked 1 or 2.

With linear estimation, the Weibull, the lognormal, the Gompertz, and the Normal are significantly better at fitting than Fisher-Pry with 95% confidence at all penetration levels. With nonlinear estimation all the models are shown to be equally good at the 5% and 10% level. At the 25%, 50%, and 75% levels, Fisher-Pry is again indicated as being significantly different. At the 75% level, the Normal is also significantly different. Comparing the fitting ability of the linear to the nonlinear version of each model, nonlinear fits are significantly better in twenty-three of the twenty-five comparisons.

If statistical fitting is the criterion used to select a forecasting model, one would expect the linear Fisher-Pry to forecast poorly compared to the other linear models, at all levels, and the other models to forecast equally well. Nonlinearly, all the models should forecast equally well up to the 10% level and Fisher-Pry would again forecast poorly at the 25% level and beyond. The nonlinear forecasts

Table 1. Model fit error (22 multi-industry cases)

LINEAR					
LEVEL	5%	10%	25%	50%	75%
1	WB(0.007)	LG(0.039)	WB(0.244)	GZ(0.822)	GZ(1.499)
2	LG(0.007)	WB(0.042)	LG(0.244)	LG(0.947)	LG(1.592)
3	GZ(0.009)	GZ(0.044)	GZ(0.255)	WB(1.037)	WB(2.028)
4	NM(0.010)	NM(0.049)	NM(0.311)	NM(1.074)	NM(2.361)
5	FP(0.015)*	FP(0.076)*	FP(0.619)*	FP(2.160)*	FP(3.747)*
E(F)	2.5	2.5	2.5	2.5	2.5
F	2.46	3.36	5.34	6.43	6.95
P>F	0.0516	0.0133	0.0007	0.0001	0.0001
LSD	0.006	0.023	0.196	0.601	0.970

NONLINEAR					
LEVEL	5%	10%	25%	50%	75%
1	WB(0.005)	LG(0.023)	EX(0.137)	EX(0.440)	LG(0.896)
2	LG(0.005)	WB(0.025)	WB(0.152)	WB(0.448)	GZ(0.950)
3	EX(0.006)	EX(0.027)	LG(0.155)	LG(0.466)	EX(1.020)
4	GZ(0.006)	GZ(0.028)	GZ(0.158)	GZ(0.494)	WB(1.055)
5	PF(0.006)	NM(0.029)	NM(0.168)	NM(0.537)	NM(1.334)*
6	NM(0.032)	FP(0.030)	FP(0.188)*	FP(0.618)*	FP(1.465)*
E(F)	2.3	2.3	2.3	2.3	2.3
F	1.01	1.48	2.45	2.31	5.84
P>F	0.4150	0.2011	0.0387	0.0491	0.0001
LSD	--	--	0.031	0.125	0.265

Key: F = calculated F, E(F) = expected F with 95% confidence, P>F = probability of a higher value of F, LSD = least significant difference, FP = Fisher/Pry, WB = Weibull, GZ = Gompertz, NM = Normal, LG = lognormal, EX = extended logistic, * = significantly different with 95% confidence

Table 2. Linear vs nonlinear fit error (22 multi-industry cases)

		FP	WB	GZ	NM	LG
5%	(1)	N(0.006)	N(0.005)	N(0.006)	L(0.010)	N(0.005)
	(2)	L(0.015)*	L(0.007)*	L(0.009)*	N(0.032)	L(0.007)*
	F	4.80	6.02	7.13	0.70	10.25
	P>F	0.0398	0.0230	0.0143	0.4108	0.0045
10%	(1)	N(0.030)	N(0.025)	N(0.028)	N(0.029)	N(0.023)
	(2)	L(0.076)*	L(0.042)*	L(0.044)*	L(0.049)*	L(0.039)*
	F	6.15	6.77	8.87	7.24	6.97
	P>F	0.0217	0.0167	0.0072	0.0137	0.0153
25%	(1)	N(0.188)	N(0.152)	N(0.158)	N(0.168)	N(0.155)
	(2)	L(0.619)*	L(0.245)*	L(0.255)*	L(0.311)*	L(0.245)*
	F	9.26	12.86	10.35	10.71	5.34
	P>F	0.0062	0.0017	0.0041	0.0036	0.0316
50%	(1)	N(0.618)	N(0.448)	N(0.494)	N(0.537)	N(0.466)
	(2)	L(2.160)*	L(1.037)*	L(0.822)*	L(1.074)*	L(0.947)*
	F	18.46	17.28	12.80	20.57	13.23
	P>F	0.0003	0.0004	0.0018	0.0002	0.0015
75%	(1)	N(1.465)	N(1.055)	L(0.822)	N(1.334)	N(0.896)
	(2)	L(3.747)*	L(2.028)*	N(0.950)	L(2.361)*	L(1.592)*
	F	20.63	17.77	0.61	21.58	10.47
	P>F	0.0002	0.0004	0.4448	0.0001	0.0040

E(F) = 4.35 for all cases

Key: L = linear, N = nonlinear, F = calculated F, E(F) = expected F with 95% confidence, P>F = probability of a higher value of F, FP = Fisher/Pry, WB = Weibull, GZ = Gompertz, NM = Normal, LG = lognormal, * = significantly different with 95% confidence should generally be better than the linear forecasts for all models and all levels, (with perhaps a little noise for the Normal at the 5% level and the Gompertz at the 75% level).

Table 3 gives the linear and nonlinear forecast error with the models ranked according to performance at each estimation level.

Table 4 shows the results of the comparison between the linear and nonlinear errors of each model at each level ranked either 1 or 2.

Of the five models studied in the linear form for forecasting ability, the Gompertz, the Normal and Fisher-Pry are statistically better than the Weibull and the Lognormal at low penetration levels. At higher penetration levels, the models cannot be distinguished statistically.

The fact that all the models studied are statistically similar at higher penetration levels has an important implication. To pick a nearly complete substitution or adoption case and then show how well a model forecasts the rest of it is an exercise in futility. At high levels of penetration most well defined models will perform well at forecasting. The differences between them will thus be insignificant.

Nonlinear estimation improves the forecasting ability of most of the models especially at high penetration levels. The extended logistic which could only be analyzed nonlinearly is accepted among the better models and, in fact, does best at the seventy-five percent level.

It is necessary at this point to relate the fitting results to the forecasting results. As noted earlier, if fitting was the determinant in deciding how well a model will forecast, one would expect:

Table 3. Model forecast error (22 multi-industry cases)

LINEAR					
LEVEL	5%	10%	25%	50%	75%
1	NM(36.4)	GZ(36.5)	NM(23.6)	NM(14.7)	NM(8.0)
2	GZ(48.5)	NM(37.4)	GZ(29.9)	GZ(16.0)	FP(8.3)
3	FP(54.3)	FP(56.7)	FP(33.4)	FP(18.8)	GZ(9.0)
4	WB(138.2)*	WB(79.6)*	WB(55.0)*	WB(19.6)	WB(9.7)
5	LG(165.1)*	LG(91.2)*	LG(65.4)*	LG(25.6)	LG(12.4)
F	9.66	3.02	4.35	1.05	0.74
E(F)	2.5	2.5	2.5	2.5	2.5
F	9.66	3.02	4.35	1.05	0.74
P>F	0.0001	0.0221	0.0030	0.3883	0.5668
LSD	53.2	40.0	24.1	--	--

NONLINEAR					
LEVEL	5%	10%	25%	50%	75%
1	GZ(51.7)	GZ(33.0)	FP(22.4)	FP(10.1)	EX(4.1)
2	NM(60.7)	NM(38.0)	NM(22.7)	GZ(10.2)	FP(4.8)
3	FP(75.1)	EX(56.0)	GZ(27.8)	NM(10.7)	WB(5.2)
4	EX(82.2)	FP(80.4)	EX(39.9)	EX(11.2)	LG(5.5)
5	WB(136.0)*	WB(84.4)	WB(49.5)*	LG(12.8)	NM(5.5)
6	LG(150.7)*	LG(85.5)	LG(58.1)*	WB(15.4)	GZ(5.7)
E(F)	2.3	2.3	2.3	2.3	2.3
F	3.50	1.95	4.69	0.91	0.59
P>F	0.0057	0.0921	0.0007	0.4747	0.7084
LSD	61.5	--	19.4	--	--

Key: F = calculated F, E(F) = expected F with 95% confidence, P>F = probability of a higher value of F, LSD = least significant difference, FP = Fisher/Pry, WB = Weibull, GZ = Gompertz, NM = Normal, LG = lognormal, EX = extended logistic.
* = significantly different with 95% confidence.

Table 4. Linear vs nonlinear forecast error (22 multi-industry cases)

		FP	WB	GZ	NM	LG
5%	(1)	L(54.3)	N(136.0)	L(48.5)	L(36.4)	N(150.7)
	(2)	N(75.1)	L(138.2)	N(51.7)	N(60.7)	L(165.1)
	F	0.87	0.08	0.15	1.89	2.17
	P>F	0.3624	0.7860	0.7068	0.1832	0.1553
10%	(1)	L(56.7)	N(84.4)	N(33.0)	L(37.4)	N(85.5)
	(2)	N(80.4)	L(79.6)	L(36.5)	N(38.0)	L(91.2)
	F	0.62	0.26	0.21	0.01	0.28
	P>F	0.4391	0.6135	0.6551	0.9327	0.5997
25%	(1)	N(22.4)	N(49.5)	N(27.8)	N(22.7)	N(58.1)
	(2)	L(33.4)*	L(55.0)	L(29.9)	L(23.6)	L(65.4)
	F	7.29	0.39	0.20	0.07	1.20
	P>F	0.0134	0.5407	0.6603	0.7882	0.2866
50%	(1)	N(10.1)	N(15.4)	N(10.2)	N(10.7)	N(12.8)
	(2)	L(18.8)*	L(19.6)	L(16.0)	L(14.7)*	L(25.6)*
	F	12.1	3.31	3.48	6.06	4.73
	P>F	0.0022	0.0830	0.0762	0.0225	0.0412
75%	(1)	N(4.8)	N(5.2)	N(5.7)	N(5.5)	N(5.5)
	(2)	L(8.3)*	L(9.8)*	L(9.0)*	L(8.0)*	L(12.4)*
	F	9.85	11.51	4.69	9.4	6.22
	P>F	0.0050	0.0027	0.0420	0.0059	0.0210

E(F) = 4.35 for all cases

Key: L = linear, N = nonlinear, F = calculated F, E(F) = expected F with 95% confidence, P>F = probability of a higher value of F, FP = Fisher/Pry, WB = Weibull, GZ = Gompertz, NM = Normal, LG = lognormal, * = significantly different with 95% confidence

- the linear Fisher-Pry to perform poorly at forecasting relative to the other models. But, in fact, Fisher-Pry is among the better models when differences between models are indicated,
- except for Fisher-Pry, the other linear models would perform equally well. But it is not so. The Weibull and the lognormal are significantly different until the 25% level,
- nonlinearly, to have no differences indicated at the 5% and the 10% level. But at the 5% level, the Weibull and the lognormal are significantly different,
- nonlinearly, only Fisher-Pry to be indicated as worse at the 25%, 50%, and 75% levels. But it is the Weibull and the lognormal, and only at the 25% level, that are indicated as significantly different,
- the nonlinear forecasts to be significantly better for all models and at all levels except maybe for the Normal at the 5% level and the Gompertz at the 75% level. But nonlinear forecasts are significantly better only at the 50% and 75% levels. In fact at the 75% level for the Gompertz, the nonlinear is better.

There seems to be overwhelming evidence therefore, not to base conclusions about the forecasting ability of growth models on their fitting ability. The process gone through in this study (i.e., empirically checking for forecasting ability) is necessary before such conclusions can be made.

These results have to be tempered with the observation that it is conceivable that slightly different conclusions could be drawn if more data were available or if a different measure of forecasting ability were used in the analysis.

The Telephone Industry Results and Discussion

As a logical extension of the analysis of the multi-industry data, it was considered worthwhile to analyze an intra-industry substitution. The telecommunication industry was the immediate contender. Data from ten telephone companies were analyzed. Only the statistically better models from the multi-industry analysis were used; namely the Gompertz, Fisher-Pry, the Normal and the extended logistic and the models were compared for forecasting ability only.

On the outset it is important to realize that this is a company by company analysis of a single substitution. Growth models have more commonly been used on a global scale (e.g., nationwide or industrywide). Global studies can be regarded as being more reliable than company by company studies because they tend to average out individual company anomalies in policy, corporate tendencies, and geographical differences. Microscopic analyses are rare in technological forecasting studies but have been done, for example, by Mansfield [30]. Moreover, many Bell operating companies have been carrying out company by company studies on the use of these models, so that the need to analyze the data on a company by company basis is justified.

From Table 5, the linear Gompertz, and Normal are significantly better than Fisher-pry. Nonlinearly, the Gompertz and Normal are again significantly better than Fisher-Pry and the extended logistic at the 5% and 10% level; and all the models perform equally well at the 25% and the 50% level.

Table 6 shows that nonlinear estimation does improve the forecasting ability in all cases except for the Gompertz at the five per cent level, although the improvement is only statistically significant for Fisher-Pry after the five percent level and for the Gompertz at the ten percent level.

These observations raise an interesting point. Recalling that Fisher-Pry was one of the better models in the multi-industry analysis, one assumes that Fisher-Pry is generally a good model and should be recommended for cases where there is hardly any indication to the contrary. But given a specific case, Fisher-Pry can easily, just like any other model, give misleading forecasts. This is the main reason for suggesting that several good models be used and the grouped forecasts used as a working range of possible outcomes.

The Use of Several Models

The fact that fitting ability is not a good indicator of forecasting ability weakens the reliability of confidence intervals in providing a working range of forecasts. The alternative is for the analyst to use several good models to provide that range. When a specific forecast as opposed to a range of forecasts is desired, it is

Table 5. Model forecast error (10 telephone company cases of electronic for electromechanical switching)

LINEAR				
LEVEL	5%	10%	25%	50%
1	GZ(14.5)	GZ(10.7)	GZ(8.2)	GZ(4.9)
2	NM(45.1)	NM(40.0)	NM(18.4)	NM(7.9)
3	FP(125.8)*	FP(112.8)*	FP(62.0)*	FP(28.1)*
E(F)	3.6	3.6	3.6	3.6
F	16.7	18.8	10.9	12.9
P>F	0.0001	0.0001	0.0008	0.0003
LSD	41.9	36.1	25.7	10.5

NONLINEAR				
LEVEL	5%	10%	25%	50%
1	GZ(14.9)	GZ(7.3)	GZ(7.1)	GZ(4.3)
2	NM(38.6)	NM(29.9)	NM(7.8)	NM(4.7)
3	EX(55.4)*	EX(46.2)*	EX(13.4)	EX(5.3)
4	FP(89.2)*	FP(71.0)*	FP(15.2)	FP(5.9)
E(F)	3.0	3.0	3.0	3.0
F	6.3	10.5	1.8	0.3
P>F	0.0022	0.0001	0.1790	0.8565
LSD	36.1	24.1	--	--

Key: F = calculated F, E(F) = expected F with 95% confidence, P>F = probability of a higher value of F, LSD = least significant difference, FP = Fisher/Pry, GZ = Gompertz, NM = Normal, EX = extended logistic
* = significantly different with 95% confidence.

Table 6. Linear vs nonlinear forecast error (10 telephone company cases of electronic for electromechanical switching)

		FP	GZ	NM
5%	(1)	N(89.2)	L(14.5)	N(38.6)
	(2)	L(125.8)	N(14.9)	L(45.1)
	F	3.9	0.02	0.3
	P>F	0.0787	0.8927	0.5973
10%	(1)	N(71.0)	N(7.3)	N(29.9)
	(2)	L(112.8)*	L(10.7)*	L(40.0)
	F	8.5	5.6	2.0
	P>F	0.0170	0.0415	0.1903
25%	(1)	N(15.2)	N(7.9)	N(7.8)
	(2)	L(62.0)*	L(8.2)	L(18.4)
	F	7.9	0.01	1.2
	P>F	0.0201	0.9087	0.2944
50%	(1)	N(5.9)	N(4.3)	N(4.7)
	(2)	L(28.1)*	L(4.9)	L(7.9)
	F	18.5	0.6	3.0
	P>F	0.0020	0.4784	0.1193

E(F) = 5.1 for all cases

Key: L = linear, N = nonlinear, F = calculated F,
E(F) = expected F with 95% confidence,
P>F = probability of a higher value of F,
FP = Fisher/Pry, GZ = Gompertz, NM = Normal,
* = significantly different with 95% confidence.

up to the analyst to decide what model to use or to use different combinations of the models and weight their forecasts subjectively or by some other means. To illustrate this concept, consider the substitution of electronic (stored program control or SPC) for electromechanical switching in the telephone industry. Figures 4-7 show the forecast decline in use of electromechanical switching as forecast by the Fisher Pry, the Gompertz and the Normal growth models with SPC having achieved the five percent, ten percent, twenty-five percent, and fifty percent penetration levels, respectively. The data used in these four examples are the aggregated data of eleven Bell operating companies.

When SPC had achieved just under five percent penetration in 1971, the three models would have given the forecasts shown in Figure 4. The actual penetration levels achieved are shown by the * symbol. If the analyst had at that time used the Fisher Pry model, his/her forecast would have been quite off. If, instead, the Gompertz had been used. the forecast would have been quite close. But in 1971, there was no way of knowing which model was predicting better. By the use of three different forecasts however, the analyst would have been able to strike a compromise, which, although possibly not as accurate as the best model, would in the long run give better forecasts than a single model used repeatedly. The concept of comparing the different model forecasts would be no different in those cases when a linear and a nonlinear version of a model were being used.

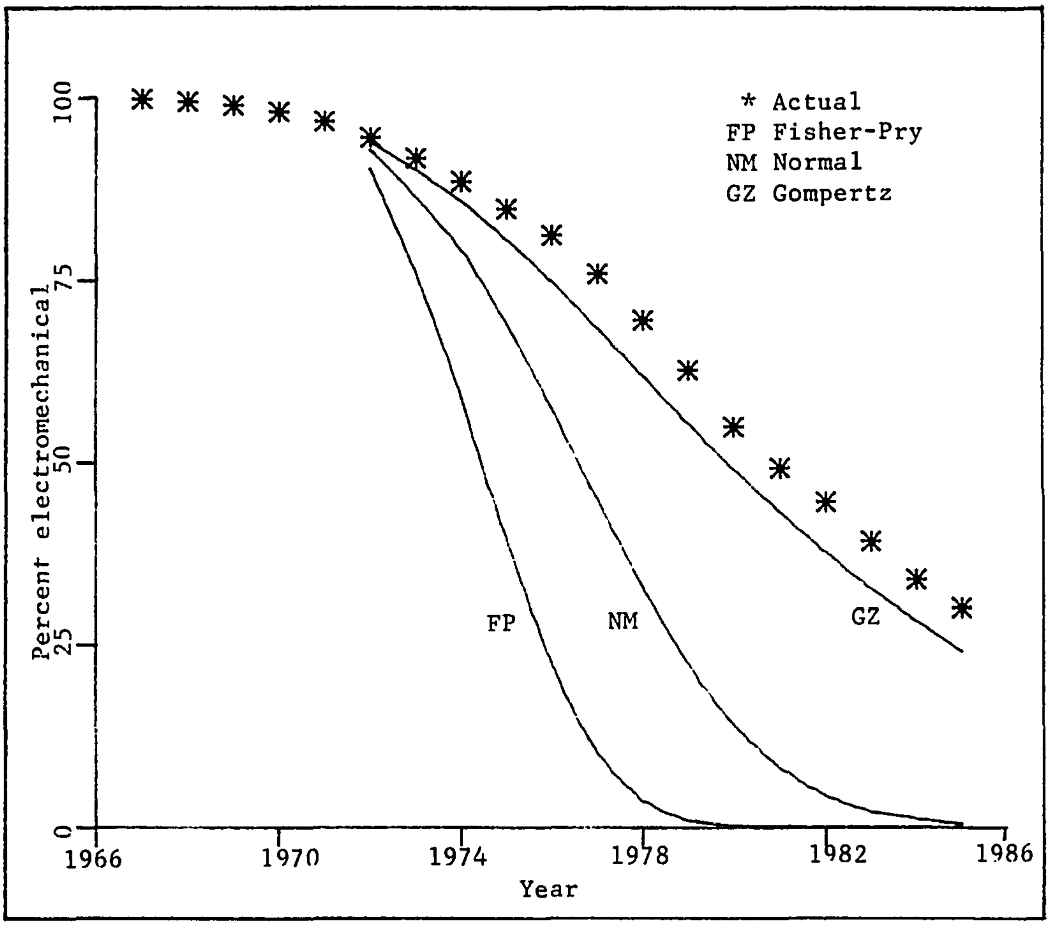

Figure 4. Decline in use of electromechanical as predicted by Fisher-Pry, Gompertz, and the Normal at five percent penetration of SPC

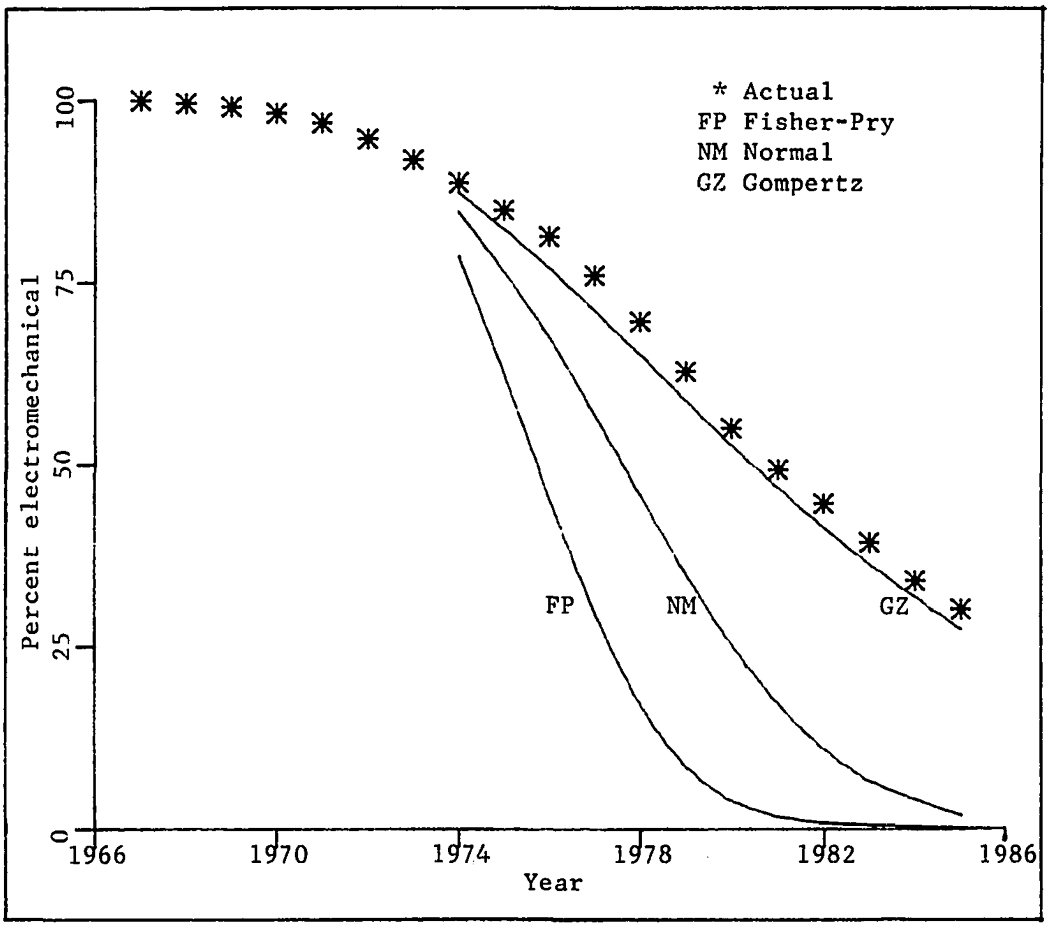

Figure 5. Decline in use of electromechanical as predicted by Fisher-Pry, Gompertz, and the Normal at ten percent penetration of SPC

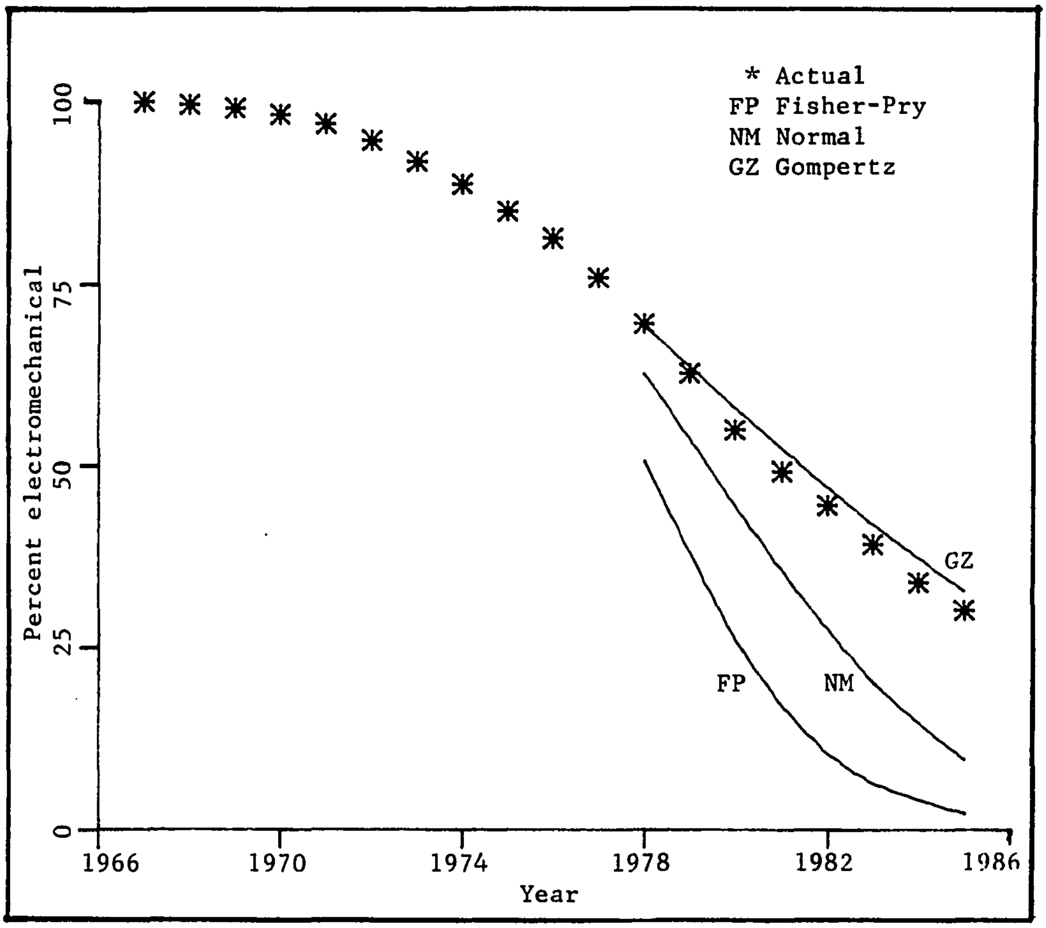

Figure 6. Decline in use of electromechanical as predicted by Fisher-Pry, Gompertz, and the Normal at twenty-five percent penetration of SPC

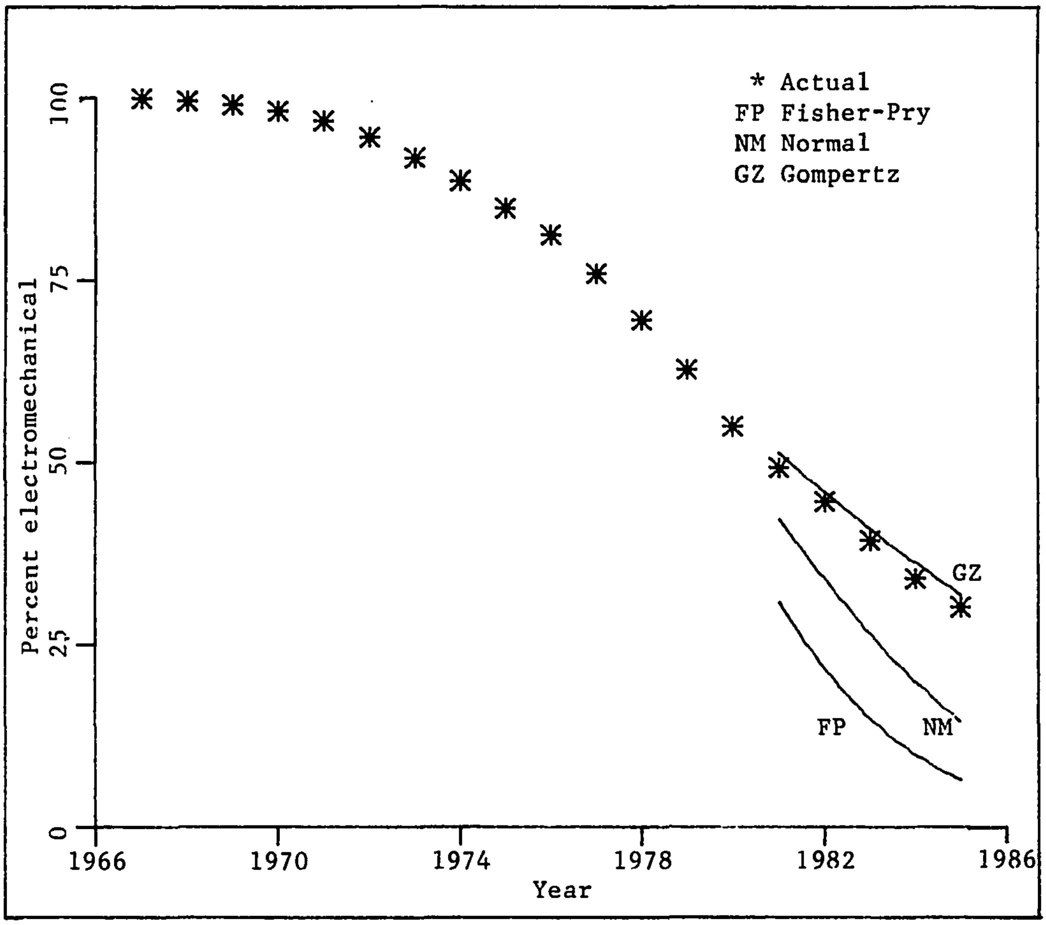

Figure 7. Decline in use of electromechanical as predicted by Fisher-Pry, Gompertz, and the Normal at fifty percent penetration of SPC

CHAPTER IX. FROM S CURVES TO LIFE CYCLES TO SERVICE LIVES

This chapter introduces the concept of combining substitution analysis predictions to obtain product life cycles and discusses the nature of life indicators that can be derived from the product life cycle. A proposal for incorporating product life cycle forecasts into the traditional analysis framework is given and demonstrated with an example.

Product Life Cycles Obtained From Growth Models

The main purpose of doing substitution analysis in life analysis and life estimation is to develop life cycles for specific types of equipment. With the life cycle furnished, life indications can then be estimated. By analyzing the different rates of substitution for the different products that provide a relatively similar service, particular life cycles for each product can be obtained. Sharif and Kabir [44] have used this approach together with dynamic programming to arrive at the life cycles.

For all practical purposes, a simpler analysis will provide the required estimates. Assume at time t_3 there are three products A, B, and C as shown in Figure 8. Originally, at t_1, there was only one product A. Then product B was introduced and has penetrated the market as shown at t_2. At t_3, product C, the newest product is encroaching on both As and Bs market share. A substitution analysis of (B+C) for A, will give the last portion of the life cycle for A. The substitution analysis of C for (A+B) will give the future progress of C. With A and

C known, B's life cycle can then be factored out. Through this kind of manipulation, any number of substitutions for one service can be handled as long as the product whose life cycle is desired is called B, those products it is substituting for are grouped as A and those products substituting for it are grouped as C. As depicted in Figure 8, the market itself might be growing (or declining) which complicates substitution analysis because a separate market size forecast has to be done.

When there are only two products, the new replacing the old, total market less the forecast new will give the life cycle of the old. The complete life cycle of the most recent product cannot be obtained by substitution analysis but it is conceivable that using standardized life cycles, (same concept as the standardized Iowa type survivor curves) a life cycle can be derived for that product too.

Life Indicators Obtained From the Product Life Cycle

According to Wolf [51], after forecasting the life cycle, it is then necessary to combine this "future rate of obsolescence" as depicted in the life cycle to yield a service life forecast. In practice however, the transposition of the information provided by the life cycle into a life estimate is a complicated exercise that requires at the very least a number of simplifying assumptions about the addition and retirement patterns in and out of the life cycle. One mathematically tenable implication of the life cycle, however, is an upper bound on any probable life forecast for the product, assuming the product life cycle

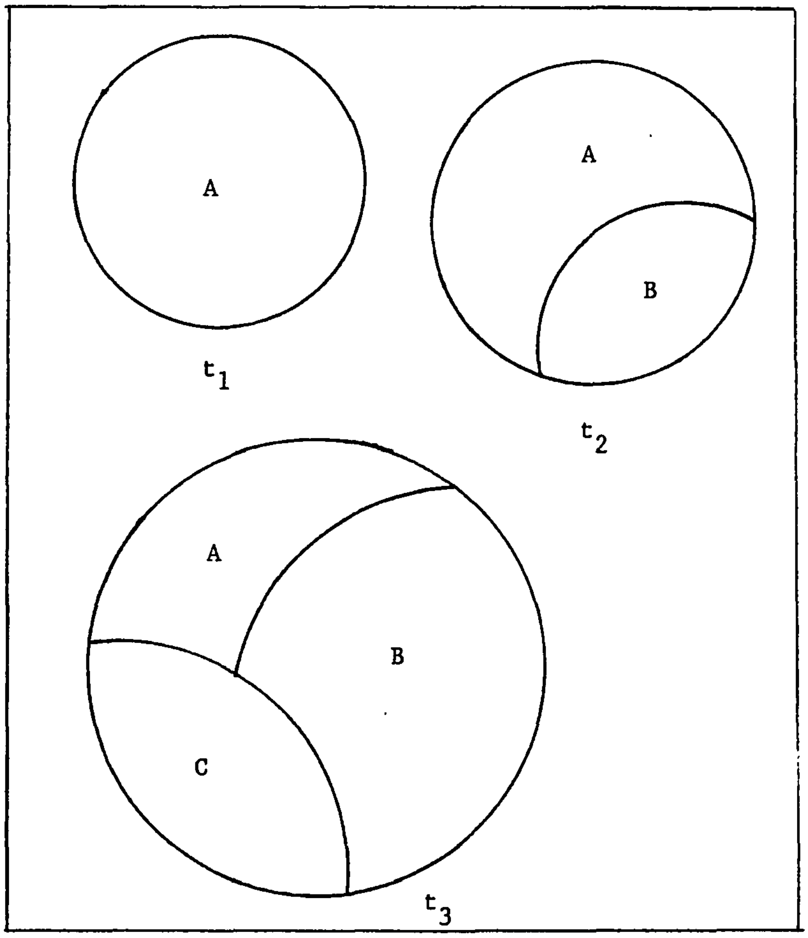

Figure 8. Progressive introduction of new products into a market

is forecast accurately. Figure 9 shows the forecast life cycle of a product with the forecast performed at time t'.

At t' there is an embedded amount of a' of the product (called the embedded balance). Various scenarios (1-4 for example) for the future experience of this embedded balance can be envisioned. Scenario 1 is the case when no retirements occur from a' until it coincides again with

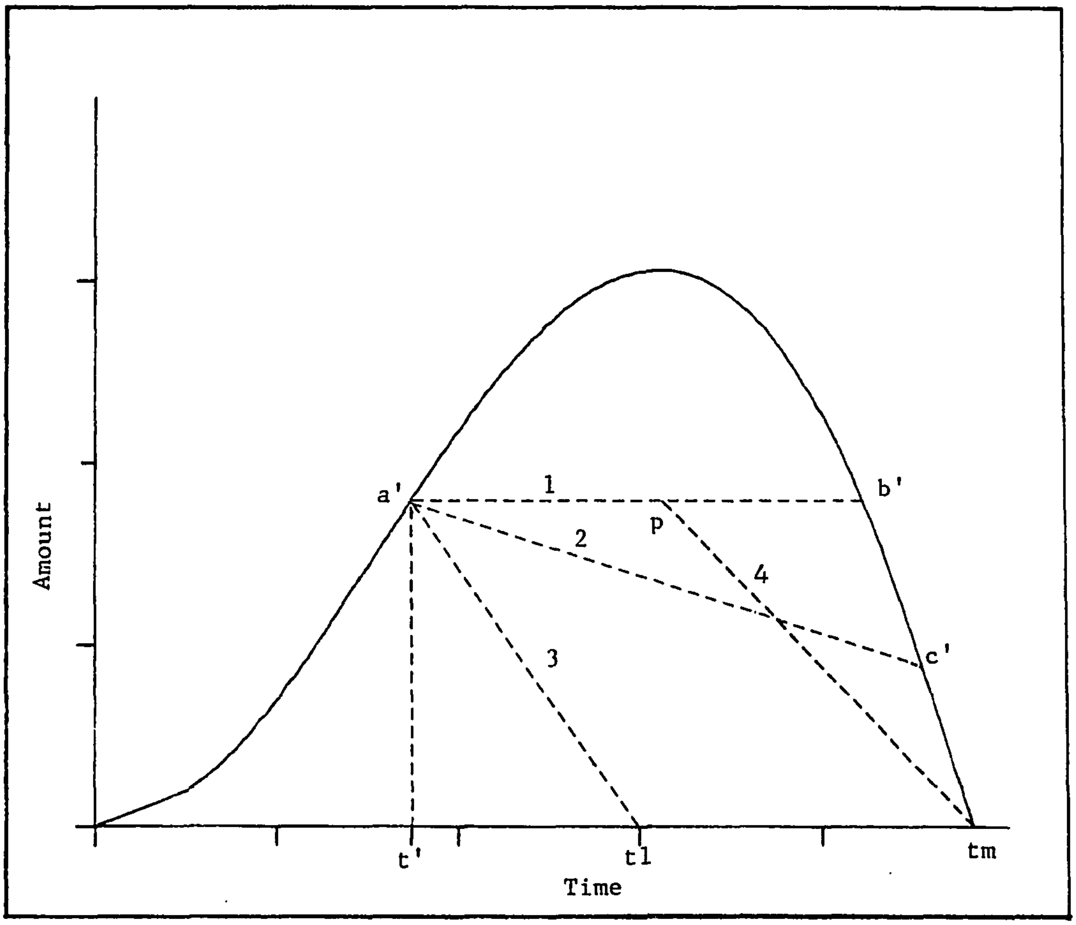

Figure 9. Different retirement patterns for an embedded balance

the amount b' on the life cycle. Then retirements start occurring so that the survivors always correspond to the amounts along the life cycle. This scenario offers the maximum remaining service to the amount a'. For this case, the remaining service for a' is the area a'b'tmt'. Obviously a'b'tmt' is an upper limit to the remaining service of a' at t' if the life cycle is forecast accurately. Scenario 2 is the situation when some retirements occur along line 2 until c' and then the embedded balance follows the life cycle. In this case, the remaining service is a'c'tmt'. For scenario 3, retirements occur from a' along line 3 so that the remaining service is a't1t'. For scenarios 2, 3, and other such scenarios, assumptions have to be made about future retirement rates if a probable remaining service is to be estimated at t'. One scenario that is intuitively appealing (scenario 4 in Figure 9) is to assume no retirements from the embedded balance until the peak of the life cycle. After the peak, the retirements are assumed to occur at the same rate as the rate for the life cycle, (and the latter can be calculated if it is assumed that there are negligible additions after the peak of the life cycle). The probable remaining service for the embedded balance would then be given by the area a'ptmt'.

For example, Figure 10 shows the life cycle of Analog electronic (SPC) switching as forecast by substitution analysis at one of the telephone companies used in this study. Table 7 gives the annual balances of this life cycle in lines.

At the end of 1985, the upper limit to any life forecast , L_u, is given by the shaded area divided by the embedded balance at the end of 1985, i.e.,

$$L_u = 5(3894)+0.5(3894)+3551+3127+......+17+0.5(14))/3894$$
$$= 11 \text{ years.}$$

If the life cycle is forecast accurately, then the actual average remaining life at the end of 1985 cannot be more than 11 years.

Table 7. Annual balances (in lines) for analog SPC at one company (1985 forecast)

Year	Lines	Year	Lines	Year	Lines	Year	Lines
1965	0	1980	2582	1995	1791	2010	90
1966	0	1981	2882	1996	1421	2011	78
1967	4	1982	3067	1997	1109	2012	65
1968	24	1983	3324	1998	855	2013	54
1969	29	1984	3757	1999	652	2014	45
1970	134	1985	3894	2000	491	2015	37
1971	197	1986	4018	2001	420	2016	31
1972	410	1987	4053	2002	358	2017	25
1973	791	1988	4153	2003	304	2018	21
1974	1043	1989	4115	2004	257	2019	17
1975	1217	1990	3895	2005	216	2020	14
1976	1411	1991	3551	2006	182		
1977	1516	1992	3127	2007	153		
1978	2017	1993	2667	2008	128		
1979	2381	1994	2212	2009	107		

A Proposal for Incorporating Substitution Analysis into Traditional Life Analysis and Estimation

In Chapter II, it was mentioned that traditional life analysis methods do not use information between accounts and that they probably complicate the analysis by aggregating time and age relationships. Although substitution analysis looks at information across accounts, it ignores age relationships and addition and retirement patterns. The information derived from either of these techniques is useful in itself but could conceivably be combined to give a better perspective on the future experience of affected equipment.

Ideally, the sum of balances from all vintages as predicted by traditional methods should, at most, be as much as the annual balance

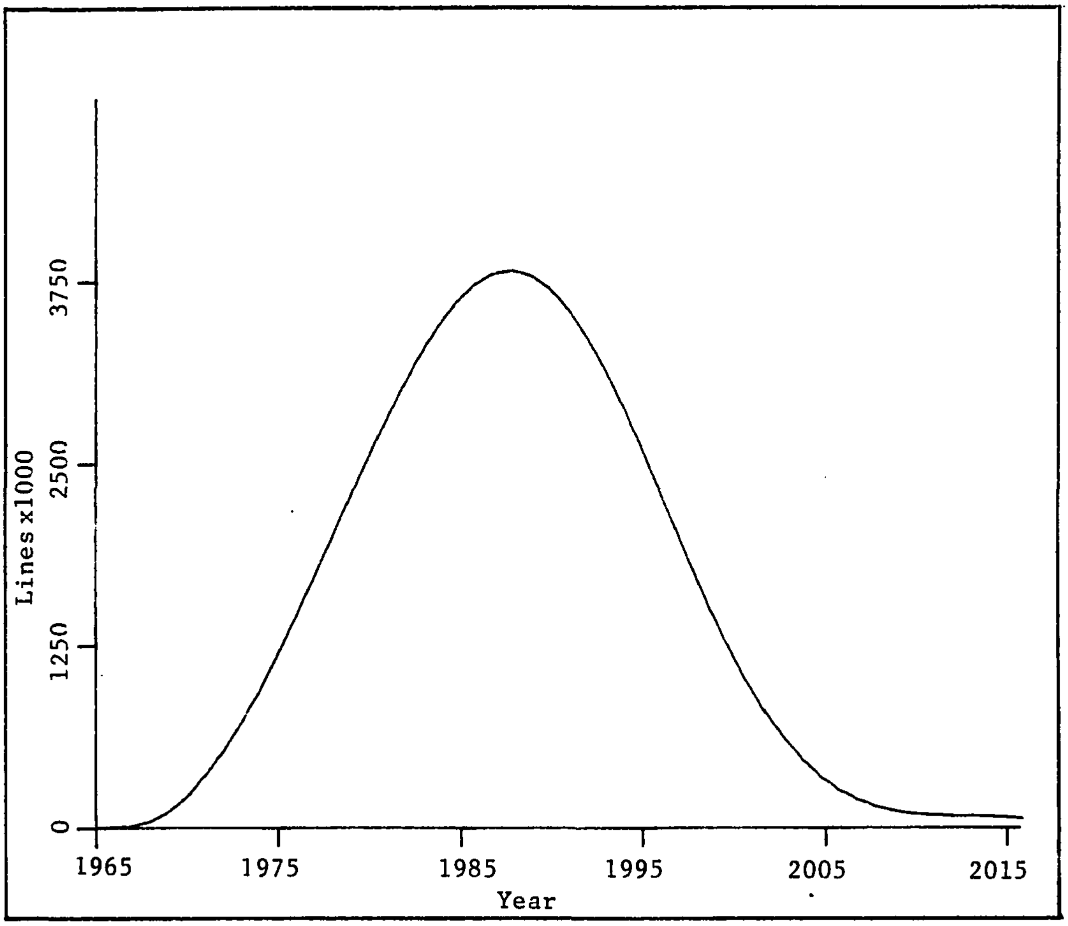

Figure 10. Forecast life cycle for analog SPC at one company (1985 forecast)

predicted by substitution analysis. Substitution analysis would give higher balances due to the effect of future additions which are not incorporated in the traditional methods.

In practice, however, there would be discrepancies. The problems facing the forecaster would then be to decide a) if those discrepancies are serious enough to warrant corrective action, and b) if corrective action is needed what form it should take. Deciding if the discrepancies are serious enough is a subjective action requiring at the very least the exercise of professional judgment in consultation with experts who have hands-on information on the nature of the account. It might be difficult to decide, let alone to prove, that the prediction provided by a particular method is the right one. However, by calculating the probable remaining life as forecast by the traditional method and comparing that life to the upper limit of life as provided by substitution analysis, a professional judgment can be made as to which method is closer to reality.

When technological advancements and competition have been driving plant experience, substitution analysis can be given the benefit of the doubt because it keeps track of what is happening in other competing accounts.

One problem is to transpose the substitution analysis information into vintage and dispersion form. Vintage analysis is necessary when more refined depreciation methods such as equal life group and remaining life are to be used.

The probable average service life (PASL) for each vintage with an original investment B_0 can be calculated as:

$$\begin{aligned} \text{PASL} &= \text{Realized Life} + \text{Unrealized Life} \\ &= \text{RL} + \text{UL} \end{aligned}$$

Then for each vintage

Total Service = PASL(B_o)

$$= \text{Realized Service} + \text{Unrealized Service}$$

$$= \quad RS \quad + \quad US$$

But

$$RS = \text{Area under historical survivor curve}$$

and

$$US = \text{Area under future portion of forecast curve}$$

so that

$$PASL = (RS + US)/B_o$$

The upper limit to probable average remaining service is the most easily defended statistic from substitution analysis. Suppose it is used as "the" probable average remaining service for the embedded balance. Suppose also that the total remaining service obtained from substitution analysis is perceived to be a better estimate than the one obtained by summing the unrealized service for all vintages from the traditional analysis. The next step, then, is to allocate the total remaining service from substitution analysis to the embedded vintages.

Different methods for performing the allocation can be envisioned. One of the most realistic is to allocate that total remaining service in direct proportion to the remaining service of each vintage as forecast by the traditional method. For example if the traditional method has forecast a remaining service of 30 \$-years for vintage V_3 and a total remaining service of 450 \$-years for all the vintages, and substitution analysis has predicted 300 \$-years for the embedded balance, the vintage V_3 would have its remaining service adjusted to 300(30/450) = 20 \$-years. In the process of making these adjustments, the original dispersions for some (or all) of the vintages might change.

An Example:

Figure 11 shows a table depicting the hypothetical experience of an actuarial account, account E656. Assume that the forecasting date is December 1975. All experience after 1975 (i.e., 1976-1982) would not be available at the forecast date. Traditional methods could then only use the available information in trying to predict the future experience. If life cycle forecasts were available however, they would provide the relevant information called "in service beginning of year" in the bottom row for the years 1976, 1977, etc. This would constitute an additional constraint to the forecasts given by traditional methods and thus improve them, if the life cycle is forecast right.

Through substitution analysis, an upper limit to probable average remaining service of 30,000 \$-years has been obtained. This upper limit is assumed to be the probable average remaining service. Assuming the 1/2 year convention, realized service for each vintage can be calculated as:

$$0.25(378)+0.75(378)+378+373+370+345+299+0.5(219) = 2252.5$$

$$0.25(392)+0.75(390)+390+390+380+350+0.5(305) = 2053.0$$

$$0.25(670)+0.75(664)+664+662+646+0.5(600) = 2937.5$$

$$0.25(690)+0.75(690)+689+680+0.5(655) = 2386.5$$

$$0.25(340)+0.75(340)+340+0.5(337) = 848.5$$

$$0.25(416)+0.75(412)+0.5(412) = 619.0$$

$$0.25(365)+0.25(365) = 182.5$$

Account E656: Special Equipment, Placements and Retirements by Calendar Years

UPPER FIGURES: Plant remaining in service at end of the indicated calendar year
LOWER FIGURES: Plant retired during indicated calendar year

Year of Placement	Plant Installed During Year	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
1969	378	378	378	373	370	345	299	219	144	91	44	31
		0	0	5	3	25	46	80	75	53	47	13
1970	392		390	390	390	380	350	305	213	132	94	63
			2	0	0	10	30	45	92	81	38	31
1971	670			664	664	662	646	600	505	365	230	150
				6	0	2	16	46	95	140	135	80
1972	690				690	689	680	655	627	495	341	226
					0	1	9	25	28	132	154	115
1973	340					340	340	337	329	299	237	189
						0	0	3	8	30	62	48
1974	416						412	412	411	401	349	311
							4	0	1	10	52	38
1975	365							365	364	358	343	319
								0	1	6	15	24
1976	355								349	348	347	330
									6	1	1	17
1977	605									603	602	598
										2	1	4
1978	710										710	708
											0	2
1979	890											890
												0
TOTALS:	In Service End of Year	378	768	1427	2114	2416	2727	2893	2942	3092	3297	3815
	Retired During Year	0	2	11	3	38	105	199	306	455	505	372

Figure 11. A hypothetical actuarial account

Assume that the traditional method has forecast the following remaining services for the vintages:

3000 \$-years for the 1969 vintage
3500 \$-years for the 1970 vintage
7500 \$-years for the 1971 vintage
8500 \$-years for the 1972 vintage
5000 \$-years for the 1973 vintage
6500 \$-years for the 1974 vintage
6000 \$-years for the 1975 vintage.
40000 \$-years Total.

The adjusted probable remaining services would be:

30000(3000/40000) = 2250 \$-years for the 1969 vintage
30000(3500/40000) = 2625 \$-years for the 1969 vintage
30000(7500/40000) = 5625 \$-years for the 1970 vintage
30000(8500/40000) = 6375 \$-years for the 1971 vintage
30000(5000/40000) = 3750 \$-years for the 1973 vintage
30000(6500/40000) = 4875 \$-years for the 1974 vintage
30000(6000/40000) = 4500 \$-years for the 1975 vintage.
Total = 30000 \$-years.

The new expectancies would then be:

2250/219 = 10.3 years for the 1969 vintage
2625/305 = 8.6 years for the 1970 vintage

5626/600 = 9.4 years for the 1971 vintage

6375/655 = 9.7 years for the 1972 vintage

3750/337 = 11.1 years for the 1973 vintage

4875/412 = 11.8 years for the 1974 vintage

4500/365 = 12.3 years for the 1975 vintage.

And the probable average service lives would be:

(2252.5 + 2250)/378 = 11.9 years for the 1969 vintage

(2053 + 2625)/392 = 11.9 years for the 1970 vintage

(2937.5 + 5625)/670 = 12.8 years for the 1971 vintage

(2386.5 + 6375)/690 = 12.7 years for the 1972 vintage

(848.5 + 3750)/340 = 13.5 years for the 1973 vintage

(619 + 4875)/416 = 13.2 years for the 1974 vintage

(182.5 + 4500)/365 = 12.8 years for the 1975 vintage.

The procedure would be no different if another mode of retirement for the embedded balance were assumed. For example, scenario 4 depicted in Figure 9 could be used to calculate an average remaining service and that estimate would then be used instead of the estimate used in the example.

In those cases (e.g., equal life group depreciation) where in addition to the vintage average service life and vintage expectancies a dispersion is also required, the procedure would be to constrain each vintage to have the calculated PASL and search for the dispersion that best satisfies its historical experience.

CHAPTER X. CONCLUSIONS AND RECOMMENDATIONS

There is a justifiable need to incorporate technological forecasting in the overall life analysis framework especially in those industries experiencing fast technological changes. Technological growth models provide a viable way of predicting future obsolescence due to technological improvements.

Of six such models studied, some models do significantly better than others, especially at low penetration levels in predicting future levels of growth, although that performance cannot easily be linked to fitting ability. The lack of a direct relationship between fitting and forecasting ability implies that fitting alone should not be used a priori to select among different models for the purposes of predicting. The models are hardly distinguishable at higher penetration levels. Nonlinear estimation improves the forecasting ability of most of the models especially at higher penetration levels.

For the telephone industry which is presently considering the use of Fisher-Pry in life analysis, it is evident that nonlinear estimation will improve the forecasting ability of the model. In addition, two other models, the Gompertz and the Normal, have been shown to predict at least as well as Fisher-Pry on an overall multi-industry basis and even better in a particular case picked from the telephone industry. It would therefore be worthwhile for the telephone industry to consider, along with Fisher-Pry, the use of these two models in their future life estimations.

In the early stages of growth it is advisable to use simpler linear estimation techniques for the models selected. As more data for a specific substitution or adoption become available, say after the 25% penetration level, a switch should be made from linear to nonlinear estimation.

After obtaining a life cycle using substitution analysis, a number of simplifying assumptions are necessary before a service life can be estimated. In all cases however, an upper limit to the average remaining life can be calculated if the life cycle is assumed to be forecast accurately. Additionally life cycle forecasts can be used as constraints on any future overall balances for an account predicted by traditional methods. A routine for doing this is proposed and demonstrated, with the upper limit to the average remaining life used as the actual average remaining life. The routine is still applicable if a retirement pattern different from the upper limit one is justified and used instead. The routine assumes that the life cycle is forecast accurately. It does not assume any dispersion pattern. When a dispersion pattern is desired, other routines, currently available, can search for the most appropriate dispersion.

Future research is necessary to resolve the different problems of incorporating technological forecasting fully into the life estimation framework, especially in regard to the assumptions of future addition and retirement patterns which the substitution and life cycle approaches do not address.

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APPENDIX A. THE DATA AND THEIR SOURCES

The abbreviation HSUS refers to: "Historical Statistics of the United States: Colonial Times to 1970." U.S. Bureau of the Census, Washington, D.C., 1975.

1. Rayon and nylon for cotton as tire cord in tire manufacture (1938-1962). Source: See Source 2.
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4. Steam and motor for sail in the United Kingdom registered shipping (1818-1938). Source: B. R. Mitchell. "Abstract of British Historical Statistics." Cambridge University Press, Cambridge, U.K., 1962, pp. 217-219.
5. Percent of underground bituminous coal automatically loaded (1923-1970). Source: "Bituminous Coal Facts." National Coal Assoc., Washington, D.C., 1972, p. 53.
6. Diesel for coal and fuel oil consumption on American railroads (1939-1970). Source: HSUS, Part II, pp. 738-739. For conversion factors from gallons to BTU's, see: ICC, "Study of Railroad Motive Power." File 66-A-11, Statement 5025, Appendix G., May 1950. Also: "America's Needs and

Resources." (See source 21, p. 757).

7. Percent of independent telephone companies connecting with the Bell system (1899-1957). Source: HSUS, Part II, pp. 783.
8. Open hearth for bessemer in raw steel production in the U.S. (1876-1960). Source: HSUS, Part II, pp. 692-693.
9. Percentage of U.S. corn acreage planted with corn hybrids (1933-1960). Source: "Agricultural Statistics." U.S. Dept. of Agriculture, Washington, D.C., various years.
10. Diesel for steam locomotives (1939-1962). Source: "Transport Statistics in the U.S." and "Statistics of Railroads in the U.S." Interstate Commerce Commission, Washington, D.C., various years.
11. Percentage of Pennsylvania anthracite mined by stripping (1927-1976). Source: "Minerals Yearbook - Mineral Fuels." 1965, Vol. II, for data 1927-1965. For data 1966-1976, "Minerals Yearbook." U.S. Bureau of Mines, Washington, D.C., various years.
12. Steam and motor for sail in the U.S, Merchant Marine (1820-1960). Source HSUS, Part II, pp. 748-750.
13. Basic oxygen process for bessemer and open hearth in raw steel production in the U.S. (1955-1981). Source: W. T. Lankford, Jr., (Ed.) "The Making, Shaping and Treatment of Steel." 10th ed. U.S. Steel, Pittsburgh, PA, 1985, p. 1508.
14. Color for B&W television in the United Kingdom (1968-1984). Source: "Annual Abstract of Statistics." Central Statistics

Office, HM's Stationery Office, London, various years.

15. Percentage of iron ore pelletized in the U.S. (1953-1973) Source: "Minerals Yearbook." U.S. Bureau of Mines, Washington, D.C., various years.

16. Percentage of farm dwelling units with electric service (1920-1956). Source: HSUS, Part II, p. 827.

17. By-product coke for oven coke in the U.S. (1900-1962). Source: E. T. Sheridan; and J. A. DeCarlo. "Coal Carbonization in the U.S.: 1900-1962." U.S. Bureau of Mines Information Circular 8251, 1965, p. 60.

18. Percentage of households in the U.S. with a television set (1946-1980). Source: "Statistical Abstract of the U.S." U.S. Bureau of the Census, Washington, D.C., various years.

19. Percentage of households in the U.S. with a color television set (1955-1984). Source: "Statistical Abstract of the U.S." U.S. Bureau of the Census, WAshington, D.C., various years.

20. Percentage of households in the U.S. with a radio receiver (1921-1970). Source: HSUS, Part II, p. 796. For number of households, see HSUS, Part I, p. 43.

21. Percentage of homes in the U.S. with at least a mechanical refrigerator (1925-1952). Source: J. F. Dewhurst; and Associates. "America's Needs and Resources: A New Survey." The Twentieth Century Fund, New York, 1955, p. 1041.

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Statistical Report." American Iron and Steel Institute, New York, various years.

APPENDIX B. MATHEMATICAL OVERVIEW

Fisher-Pry

The model is a behavioral formulation and it assumes that the proportional rate of increase of the market for the new product is directly proportional to the amount still to be substituted, i.e.:

$$\frac{1}{y}\frac{dy}{dt} = k(1-y) \tag{1}$$

where k is a proportionality constant, y is the fractional share of the new product, and t is time. Integrating (1) gives

$$\frac{y}{1-y} = e^{k(t-t')} \tag{2}$$

where t' is an integration constant and is defined in this case as the time when y=0.5. Equation (2) can be reduced further to give

$$y = \frac{1}{1+e^{-k(t-t')}} \tag{3}$$

which is the logistic equation with an upper limit of unity. Taking logarithms of equation (2), one obtains the Fisher-Pry model as

$$\ln \frac{y}{1-y} = b(t-t')$$

Obviously, by assuming an upper limit of unity for the logistic and linearizing with logarithms, one again obtains Fisher-Pry. Differentiating (3) twice with respect to t gives the inflection point at t=t', where, as noted earlier, y=0.5.

To show that the Fisher-Pry curve is symmetric, it is easier to work with (1) in the form

$$\frac{dy}{dt} = k(1-y)y \tag{4}$$

with y as defined in (3). Shifting time so that t=0 coincides with t' when y=0.5, one gets

$$y = \frac{1}{1+e^{-kt}}$$

then (4) reduces to

$$\frac{dy}{dt} = \frac{ke^{-kt}}{(1+e^{-kt})^2}$$

If dy/dt is symmetric about t', one would expect dy/dt to be unchanged with changes in the sign of t so that

$$\frac{ke^{-kt}}{(1+e^{-kt})^2} = \frac{ke^{kt}}{(1+e^{kt})^2}$$

or

$$\frac{ke^{-kt}(1+2e^{kt}+e^{2kt})}{ke^{kt}(1+2e^{-kt}+e^{-2kt})}$$

to be equal to 1, which is the case.

The Gompertz

If R(t), a function of time, is defined as the rate of growth in y so that in a time interval dt

$$dy = R(t)ydt \tag{5}$$

If, further, the change in R(t) is assumed to be proportional to R(t), and noting that R(t) is a monotonically decreasing function, then

$$\frac{dR(t)}{dt} = -kR(t)$$

which on integrating gives

$$\ln R(t) = kt + C$$

where C is a constant of integration. Then

$$R(t) = e^{-kt+C}$$
$$= Ae^{-kt}$$

where A is exp(C). But from (5)

$$\frac{dy}{y} = R(t)dt$$
$$= Ae^{-kt}dt$$

which on integrating gives

$$\ln y = \frac{-A}{k} e^{-kt} + K$$

where K is an integration constant. Thus

$$\ln y = -Ge^{-kt} + K$$

where G=-A/k so that

$$y = e^{-Ge^{-kt}+K}$$

But as t goes to infinity, y tends to an upper limit L, so that

$$L = e^{-G(0)+K}$$
$$= e^{K}$$

which leads to

$$y = Le^{-Ge^{-kt}}, \qquad (6)$$

which is the equation for the Gompertz growth function. Differentiating (6) twice with respect to t gives the inflection point of the Gompertz at

$$t = \frac{1}{k} \ln G$$

where y = L/e and e is the base of the natural log so that the inflection point occurs at approximately 37 percent. Taking the logarithms of (6) twice gives

$$\ln \ln \frac{1}{y} = \ln G - kt$$

which is the linear form of the Gompertz.

The Weibull

The 3-parameter Weibull growth curve is given as

$$y = 1 - e^{-\left(\frac{t-\mu}{\eta}\right)^{\beta}} \tag{7}$$

where μ is a threshold or shift parameter before which no substitution would have taken place. The parameters β and η are the shape and scale parameter respectively. When β is 1, the Weibull reduces to the negative exponential distribution. When β is 2, it reduces to the Rayleigh distribution and at values of $3<\beta<4$ it approximates the normal distribution. For very large values of β, e.g. $\beta>12$, it approximates the smallest extreme value distribution. The parameter η is the (e-1/e)th or approximately the 63rd percentile of the curve.

Taking the logarithm of (7) twice reduces the function to its linear form as

$$\ln \ln \frac{1}{1-y} = \beta \ln(t-\mu) - \beta \ln \eta$$

The inflection point occurs at

$$t = \mu + \eta \sqrt[\beta]{\frac{\beta-1}{\beta}}$$

at which time

$$y = 1 - e^{-\left(\frac{\beta-1}{\beta}\right)}$$

Because the Weibull is a three parameter model, the estimation of its parameters is more complicated than for a two parameter model. The procedure for estimating its threshold parameter is discussed in the body of the dissertation.

The Normal

The normal growth curve is an analog of the statistical normal cumulative distribution function

$$y(t) = \int_{-\infty}^{t} (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right) dt \qquad (8)$$

whose standard normal cumulative distribution function is

$$\Phi(Z) = \int_{-\infty}^{Z} (2\pi)^{-\frac{1}{2}} \exp(-t^2/2)\, dt \qquad (9)$$

and is tabulated in many statistical texts. Its inflection point occurs at time $t = \mu$. Linear estimation is achieved by rewriting (8) as

$$y(t) = \Phi[(t-\mu)/\sigma]$$

or

$$t = \mu + \sigma\Phi^{-1}[y(t)]$$

so that a plot of the observed time versus the standard normal variate having the observed percentile should plot as a straight line with an intercept of μ and a slope of σ.

The Lognormal

The lognormal growth curve is derived from its statistical analog as:

$$y(t) = \int_0^t (2\pi\sigma^2 t^2)^{-\frac{1}{2}} \exp(-(\log(t-\tau)-\mu)^2/2\sigma^2)\ dt$$

The standard normal cumulative distribution can be used to linearize the lognormal if it is represented in the form

$$y(t) = \Phi[(\log(t-\tau)-\mu)/\sigma]$$

where Φ[ ] is the standard normal cumulative distribution function. As for the normal, plotting log (t-τ) versus the standard normal variate having the observed percentile should result in a straight line with an intercept μ and a slope σ. Like the Weibull, the lognormal is a three parameter model with a threshold parameter whose estimation is discussed in the body of the dissertation.

The Extended Logistic

Bass derived a behavioral model by analyzing the probabilistic behavior of the adopters of a new technology. He distinguished between innovators and imitators and arrived at the probability of adoption at time t as

$$p(t) = p + \left(\frac{q}{m}\right)y(t)$$

where p and q are called coefficients of innovativeness and imitativeness respectively and m is the maximum adoptions to be achieved. A little mathematical manipulation leads to

$$y(t) = \frac{m(1-e^{-(p+q)t}}{1+\left(\frac{q}{p}\right)e^{-(p+q)t}}$$

as the adoptions achieved at time t. This is the form of the model as proposed by Bass. Mahajan et al. [28] noted that when p=0 (no innovators), the adoption does not take place. To correct this difficulty, they introduced an existing level of adoption a at time t=0 so that

$$y(t) = \frac{m-pZ(t)}{1+\left(\frac{q}{m}\right)Z(t)}$$

where

$$Z(t) = \frac{(m-a)e^{-(p+q)t}}{p + \frac{qa}{m}}$$

Meade [34] renamed this curve the extended logistic. No easy way of linearizing it could be found and for the purposes of this study, it was used in its original form (nonlinearly).

APPENDIX C. TABLES OF CASE BY CASE ERRORS FOR EACH MODEL

Table 8. Mean estimate error for each linear model at 5% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	0.01223	0.00143	0.00554	0.00797	0.00032
2	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.10225	0.00273	0.02912	0.04949	0.00222
4	0.00933	0.00095	0.00129	0.00320	0.00081
5	0.00665	0.01614	0.01441	0.01112	0.01966
6	0.00000	0.00000	0.00000	0.00000	0.00000
7	0.01533	0.02274	0.02103	0.01875	0.02468
8	0.00012	0.00000	0.00010	0.00011	0.00000
9	0.00103	0.00020	0.00080	0.00010	0.00155
10	0.00330	0.00055	0.00152	0.00219	0.00046
11	0.01346	0.01374	0.01348	0.01347	0.01374
12	0.02823	0.03025	0.02826	0.02822	0.03020
13	0.01881	0.01116	0.00644	0.00986	0.00819
14	0.02155	0.00037	0.00145	0.00584	0.00033
15	0.01334	0.00950	0.01214	0.01263	0.00888
16	0.00108	0.00065	0.00069	0.00081	0.00067
17	0.01474	0.01931	0.01975	0.01710	0.02293
18	0.00000	0.00000	0.00000	0.00000	0.00000
19	0.01814	0.00001	0.00368	0.00748	0.00000
20	0.01990	0.00003	0.00185	0.00636	0.00001
21	0.01191	0.01325	0.01291	0.01243	0.01382
22	0.01513	0.01565	0.01459	0.01471	0.01647

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 9. Mean estimate error for each linear model at 10% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	0.03455	0.00112	0.00969	0.01756	0.00098
2	0.22212	0.05602	0.11435	0.15423	0.03119
3	0.13494	0.07052	0.02825	0.04692	0.05191
4	0.01022	0.00433	0.00435	0.00279	0.00417
5	0.05459	0.09610	0.09189	0.07734	0.11276
6	0.02740	0.01384	0.01666	0.02091	0.01243
7	0.06476	0.10022	0.09483	0.08332	0.11239
8	0.13344	0.07760	0.12145	0.12682	0.07547
9	0.00890	0.00064	0.01500	0.00250	0.01922
10	0.01448	0.00446	0.00415	0.00665	0.00446
11	0.06503	0.07467	0.07042	0.06766	0.07755
12	0.11721	0.07007	0.07691	0.09187	0.06153
13	0.04631	0.00840	0.00575	0.01227	0.00592
14	0.05270	0.02054	0.01784	0.00447	0.01477
15	0.12828	0.12245	0.10963	0.11414	0.11736
16	0.00476	0.01692	0.01237	0.00855	0.02077
17	0.47176	0.13728	0.12028	0.20695	0.08048
18	0.00806	0.00119	0.00441	0.00004	0.00343
19	0.01923	0.02998	0.02165	0.00978	0.02229
20	0.01990	0.00003	0.00185	0.00636	0.00001
21	0.00645	0.01015	0.01051	0.00790	0.01416
22	0.02290	0.01247	0.00976	0.01243	0.01228

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 10. Mean estimate error for each linear model at 25% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	0.36894	0.11748	0.12080	0.18169	0.11230
2	0.26045	0.63486	0.38305	0.30765	0.50893
3	0.21047	0.29173	0.09414	0.07024	0.17243
4	1.68377	0.29564	0.55596	0.89499	0.24497
5	0.12630	0.40429	0.45140	0.31385	0.57396
6	0.12841	0.01986	0.06331	0.08977	0.01801
7	0.04964	0.12332	0.16993	0.08640	0.22784
8	0.17978	0.30124	0.24008	0.20555	0.33564
9	0.91505	0.11421	0.01855	0.09459	0.03317
10	0.02982	0.04876	0.05904	0.03263	0.10138
11	0.32704	0.21875	0.24372	0.27191	0.22761
12	0.95822	0.37144	0.54617	0.70618	0.28684
13	0.04022	0.03468	0.11473	0.04276	0.15549
14	0.26551	0.01111	0.06961	0.00364	0.09996
15	2.48735	1.32694	1.00619	1.51569	0.93153
16	0.21170	0.19522	0.19457	0.18901	0.20191
17	2.52929	0.29992	0.30349	0.80462	0.11245
18	0.27331	0.08930	0.04157	0.00586	0.00100
19	0.30120	0.23324	0.67475	0.34771	0.80432
20	1.97013	0.08827	0.09416	0.50468	0.00239
21	0.27502	0.09631	0.02963	0.09541	0.04778
22	0.03083	0.06527	0.13916	0.07801	0.18885

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 11. Mean estimate error for each linear model at 50% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	3.69468	1.67305	1.08279	2.00519	1.20272
2	4.07384	1.41570	1.44985	2.66909	1.10077
3	0.21965	1.29555	1.23263	0.61307	2.28072
4	1.10743	0.96353	0.77732	0.55774	1.04970
5	0.39657	1.38213	2.16547	1.27629	2.54544
6	1.08283	1.60293	1.93794	1.46003	2.06662
7	1.84549	0.94892	0.10868	0.33825	0.17309
8	0.32269	0.86995	0.82901	0.50241	1.20729
9	4.82002	0.77352	0.08191	1.23534	0.12517
10	0.14765	0.62160	0.90519	0.45771	1.02830
11	0.63931	0.30416	0.28109	0.39563	0.30447
12	1.03151	0.85104	0.76488	0.76680	1.23058
13	0.36282	0.20817	0.78819	0.20905	0.85627
14	4.41249	0.94244	0.24572	1.23453	0.36699
15	6.04469	2.17976	1.58830	3.52949	0.79015
16	0.28037	0.32029	0.82316	0.44995	1.13105
17	3.02609	0.25029	0.25273	0.73056	0.38347
18	4.94099	3.52561	0.43755	1.95576	0.60065
19	2.59713	0.90887	0.65814	0.36074	0.72257
20	5.70878	1.17618	0.62487	2.26092	0.13688
21	0.68464	0.11078	0.13025	0.22671	0.11706
22	0.08743	0.49026	0.91858	0.39024	1.42284

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 12. Mean estimate error for each linear model at 75% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	7.15601	3.67231	2.36336	5.08356	2.30816
2	4.08234	1.24735	1.25270	2.71546	0.98233
3	0.28249	0.65994	2.77837	0.89943	4.49304
4	1.72684	0.85764	0.67516	0.80375	0.61826
5	0.38282	1.26106	2.52772	1.08352	2.63202
6	3.28521	3.04221	1.90938	2.85689	2.04058
7	4.45628	1.90795	0.47941	1.83872	0.50696
8	0.57547	1.51720	2.31806	0.97209	2.73625
9	8.26574	1.86954	0.56065	4.14841	0.27649
10	0.32227	0.99563	2.71886	1.01341	2.36689
11	0.62607	0.74769	0.90848	0.52845	1.45463
12	0.84037	1.23293	1.51619	0.79464	2.69426
13	2.75080	1.95286	0.87627	1.37347	1.04116
14	11.04635	3.70980	1.47395	6.31869	0.82711
15	4.48695	0.95608	1.23344	2.70879	1.06916
16	0.60328	1.62302	2.39403	1.11331	2.72779
17	2.30497	0.77551	0.94895	0.74231	1.27346
18	6.47256	5.79164	0.55576	3.48190	0.59806
19	4.99639	2.33924	0.52712	0.91095	0.47842
20	8.20880	2.82317	1.44659	5.13221	0.62006
21	6.03430	3.03666	2.26356	4.87011	1.86512
22	3.53813	2.59840	1.24465	2.54927	1.42284

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 13. Mean estimate error for each nonlinear model at 5% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	0.00519	0.00059	0.00323	0.00407	0.00016	0.00000
2	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.01436	0.00245	0.00940	0.01151	0.00212	0.00336
4	0.00266	0.00085	0.00102	0.00157	0.00080	0.00108
5	0.00190	0.00343	0.00510	0.00348	0.00000	0.00298
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
7	0.00677	0.00858	0.01074	0.00888	0.01065	0.00962
8	0.00012	0.00000	0.00010	0.00011	0.00000	0.00006
9	0.00003	0.00009	0.00024	0.00008	0.00030	0.00003
10	0.00236	0.00050	0.00130	0.00170	0.00040	0.00040
11	0.01330	0.01360	0.01340	0.01340	0.01360	0.01330
12	0.02470	0.02660	0.02620	0.02550	0.02740	0.03130
13	0.00900	0.00710	0.00570	0.59120	0.00640	0.00640
14	0.00100	0.00000	0.00010	0.00040	0.00010	0.00000
15	0.01120	0.00810	0.01090	0.01100	0.00780	0.01120
16	0.00090	0.00060	0.00060	0.00070	0.00060	0.00060
17	0.01380	0.01420	0.01530	0.01450	0.01520	0.01380
18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
19	0.00060	0.00000	0.00020	0.00040	0.00000	0.00000
20	0.00170	0.00000	0.00030	0.00080	0.00000	0.00000
21	0.00930	0.01020	0.01120	0.01030	0.01150	0.01410
22	0.01410	0.01550	0.01430	0.01420	0.01630	0.01510

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 14. Mean estimate error for each nonlinear model at 10% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	0.00866	0.00050	0.00414	0.00600	0.00085	0.00026
2	0.09964	0.03203	0.07615	0.08714	0.02410	0.02804
3	0.02176	0.03142	0.02279	0.02124	0.03635	0.02059
4	0.00244	0.00326	0.00373	0.00255	0.00354	0.00217
5	0.00855	0.01170	0.02132	0.01502	0.01896	0.02614
6	0.02266	0.01327	0.01583	0.01882	0.01223	0.01293
7	0.00699	0.00872	0.01466	0.01087	0.01287	0.02639
8	0.12262	0.07674	0.11688	0.11968	0.07511	0.11739
9	0.00015	0.03400	0.00165	0.00032	0.00128	0.00004
10	0.00720	0.00420	0.00380	0.00480	0.00430	0.00350
11	0.04930	0.05430	0.06000	0.05470	0.06160	0.07830
12	0.09160	0.06710	0.07260	0.08090	0.06040	0.07900
13	0.00840	0.00530	0.00540	0.00590	0.00530	0.00520
14	0.00120	0.00510	0.00790	0.00360	0.00640	0.00160
15	0.11890	0.11020	0.10200	0.10850	0.10230	0.11190
16	0.00360	0.01020	0.00970	0.00640	0.01420	0.00390
17	0.07210	0.04790	0.05500	0.06240	0.04390	0.04560
18	0.00050	0.00000	0.00060	0.00000	0.00030	0.01130
19	0.00400	0.00640	0.00820	0.00610	0.00750	0.00410
20	0.00170	0.00000	0.00030	0.00080	0.00000	0.00000
21	0.00600	0.00640	0.00780	0.00620	0.00860	0.01030
22	0.01190	0.01140	0.00940	0.00980	0.01160	0.01010

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 15. Mean estimate error for each nonlinear model at 25% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	0.17172	0.11610	0.11714	0.13943	0.11172	0.12803
2	0.25328	0.44676	0.37137	0.30447	0.44663	0.28733
3	0.04892	0.09575	0.08601	0.06224	0.10790	0.05523
4	0.42935	0.23439	0.33376	0.37954	0.22205	0.23927
5	0.02309	0.02491	0.05751	0.03458	0.04469	0.04529
6	0.08111	0.01890	0.05247	0.06608	0.01751	0.01893
7	0.03664	0.02826	0.02132	0.02267	0.02061	0.03423
8	0.17434	0.24898	0.22901	0.19564	0.29211	0.18480
9	0.07383	0.03400	0.00857	0.03185	0.01074	0.04826
10	0.02259	0.03867	0.05120	0.03184	0.06843	0.02663
11	0.24487	0.21655	0.22795	0.23454	0.22718	0.20911
12	0.52367	0.29199	0.43203	0.47835	0.26383	0.28063
13	0.01403	0.02305	0.05725	0.03003	0.05383	0.01482
14	0.00798	0.00363	0.01041	0.00308	0.00842	0.00510
15	1.17363	0.89379	0.86023	1.01999	0.79632	0.83210
16	0.18272	0.19311	0.19455	0.18467	0.20032	0.18008
17	0.18230	0.08134	0.10540	0.13786	0.07940	0.09034
18	0.01791	0.00655	0.00438	0.00135	0.00000	0.01697
19	0.20226	0.20152	0.11591	0.15545	0.13568	0.20063
20	0.11430	0.00634	0.01579	0.05523	0.00199	0.01917
21	0.11811	0.09743	0.02930	0.06554	0.04143	0.06258
22	0.02879	0.04241	0.09400	0.05447	0.09996	0.03379

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 16. Mean estimate error for each nonlinear model at 50% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	1.89527	1.38847	1.06404	1.55710	1.16668	1.45342
2	2.24166	1.26091	1.33497	1.90109	1.07770	1.34549
3	0.17174	0.26004	0.48914	0.28441	0.46731	0.18278
4	0.41338	0.66593	0.74881	0.48890	0.85253	0.46019
5	0.16472	0.19733	0.26835	0.17350	0.19381	0.22524
6	0.66565	0.74674	1.40013	0.92781	1.19329	0.99322
7	0.30886	0.22121	0.05680	0.15651	0.06762	0.21260
8	0.23358	0.38010	0.62370	0.34495	0.67325	0.26433
9	0.47547	0.15662	0.05086	0.25576	0.06128	0.25570
10	0.08252	0.15011	0.45241	0.20015	0.35493	0.09378
11	0.37720	0.29184	0.27241	0.31007	0.30238	0.25765
12	0.73974	0.78736	0.76474	0.70831	0.99615	0.69896
13	0.17669	0.19204	0.14506	0.12415	0.12035	0.16473
14	0.83646	0.48857	0.22998	0.56123	0.31540	0.61089
15	2.00264	0.91039	1.09405	1.65519	0.65903	0.89564
16	0.27636	0.29230	0.57357	0.35079	0.56699	0.28162
17	0.18395	0.19716	0.21884	0.15271	0.32041	0.13161
18	0.95216	0.56649	0.27833	0.64789	0.28512	0.41813
19	0.43791	0.40688	0.15551	0.29869	0.21598	0.41648
20	0.70447	0.12258	0.21328	0.47360	0.05137	0.16615
21	0.21217	0.09743	0.12973	0.12882	0.11680	0.10487
22	0.05371	0.08530	0.29866	0.11265	0.20336	0.05455

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 17. Mean estimate error for each nonlinear model at 75% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	3.96851	2.83688	2.26093	3.64706	2.22284	2.90628
2	2.06589	1.01120	1.10956	1.73805	0.95165	1.06273
3	0.27790	0.40531	0.56840	0.33964	0.46192	0.28309
4	0.68669	0.56791	0.58931	0.58864	0.54912	0.52838
5	0.28193	0.38526	0.26485	0.22547	0.20763	0.28187
6	2.69503	2.73090	1.61057	2.61596	1.73279	2.36854
7	1.11745	0.70552	0.47926	0.80574	0.48141	0.79469
8	0.36401	0.50588	1.43925	0.57468	1.19625	0.64158
9	1.63225	0.50203	0.25676	1.24596	0.14102	0.77515
10	0.11640	0.15311	0.94761	0.30011	0.54264	0.24486
11	0.50772	0.66738	0.86988	0.51208	1.16804	0.56292
12	0.72212	0.96118	1.36977	0.78734	1.73185	0.78422
13	1.32588	1.32911	0.45545	1.10942	0.59647	1.14176
14	2.91608	1.32691	0.91838	2.47960	0.60916	1.65291
15	1.60513	0.87802	0.99680	1.33732	1.01953	0.88738
16	0.49338	0.57671	1.42866	0.69697	1.16215	0.53551
17	0.42943	0.61079	0.85980	0.48377	0.91917	0.44017
18	1.51362	0.84561	0.26211	1.15273	0.20628	0.43859
19	0.62513	0.56415	0.13761	0.41032	0.21564	0.56149
20	2.67443	1.02232	0.99575	2.40452	0.61537	1.13713
21	3.91492	2.51096	2.10141	3.77781	1.82276	2.55746
22	2.30110	2.10834	0.98811	2.11850	1.15489	1.85352

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 18. Mean forecast error for each linear model at 5% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	58.09556	98.57732	4.31873	23.16103	267.81863
2	134.81519	24.30134	35.49505	97.82488	149.91426
3	33.30774	267.21398	62.65787	1.65759	300.10642
4	123.42923	9.55070	10.93494	70.69820	4.17121
5	16.89433	120.61390	218.99180	100.78769	276.86668
6	6.02906	521.68769	83.15135	24.74025	531.13550
7	25.40605	15.61692	62.06360	8.29616	97.75066
8	1.75913	416.66890	44.44450	5.05019	424.73315
9	74.31581	47.41187	2.52112	34.53763	3.80340
10	15.07202	153.13457	56.83328	3.56778	208.35089
11	45.44527	144.85423	91.12372	72.49363	148.47649
12	24.55502	95.61741	34.39118	1.71081	159.59208
13	39.54138	26.22255	29.24606	7.40511	5.79397
14	97.92285	21.68553	4.12606	42.73454	75.82086
15	20.80167	197.30061	67.76033	18.43007	207.62381
16	5.68538	103.90214	59.01857	14.83374	124.55228
17	224.02410	196.49877	42.32525	149.39621	49.23887
18	41.46435	24.27319	1.07786	19.72554	4.86007
19	35.32534	285.83392	96.81357	8.56600	308.60281
20	100.59241	32.05661	28.28627	72.48916	1.38559
21	37.33401	20.70669	11.83422	13.10810	26.05351
22	31.81247	215.95429	18.97663	9.93464	255.67447

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 19. Mean forecast error for each linear model at 10% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	34.68012	116.03302	13.23077	10.52225	272.23040
2	54.24392	56.68350	1.42835	27.33535	158.31862
3	6.71130	273.71311	86.74827	11.51345	274.77353
4	120.26839	51.87994	14.09382	72.59090	22.47071
5	7.20170	48.79048	170.59561	62.58687	200.58675
6	5.24574	19.57848	14.42342	2.13045	78.59138
7	7.95355	22.46796	23.63020	5.84490	27.77448
8	9.31310	383.98240	87.04122	30.36104	401.24373
9	72.15424	62.75347	4.10826	38.99803	7.68848
10	1.15629	132.53527	93.30118	24.52016	164.05175
11	164.15605	119.81236	34.01131	114.59725	14.29224
12	113.77068	9.85831	12.89699	64.85554	43.45000
13	26.30389	10.58164	35.56457	7.37667	25.60757
14	91.32118	29.84745	3.57148	46.03191	9.86596
15	126.66916	103.73183	12.21267	78.40716	12.50804
16	13.04621	38.26698	40.22287	6.21765	86.73865
17	156.36249	47.74647	11.13502	79.38846	22.30073
18	36.42292	36.83144	2.38327	21.07991	7.62073
19	29.59886	41.62359	74.55586	7.20216	64.52325
20	100.59241	32.05661	28.28627	72.48916	1.38559
21	54.91575	34.07079	2.54374	28.08547	5.07239
22	14.71164	77.67972	36.30523	10.81809	106.22552

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 20. Mean forecast error for each linear model at 25% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	25.39819	5.42179	8.71040	10.26373	17.32499
2	43.92954	29.50974	1.90880	25.62228	18.36916
3	0.29476	202.55183	95.04571	21.45565	171.59965
4	27.63616	39.83093	5.24030	9.27833	107.69931
5	2.43081	12.63477	98.81927	24.99178	95.44509
6	5.10399	179.37421	45.33261	12.90011	235.20410
7	62.76660	49.17342	3.05644	20.67254	1.33065
8	6.29317	68.95488	47.64945	14.84105	125.62863
9	59.40032	47.38111	6.89857	37.83266	10.04211
10	7.94355	57.44752	101.58326	36.46598	152.94824
11	22.05349	31.03333	6.05900	5.61586	53.12285
12	5.50591	93.18718	38.48395	4.36716	147.04087
13	25.42690	18.37984	21.70001	8.50272	29.35481
14	86.38300	56.45686	5.18521	49.95676	6.55910
15	105.15741	66.28688	13.73255	70.81661	9.31561
16	13.62562	85.97385	86.39044	42.32022	103.39515
17	44.43567	10.38423	19.20125	9.24522	84.80271
18	30.77260	32.12811	4.45094	20.86707	18.39224
19	39.18524	42.29786	12.04986	9.52623	6.42434
20	59.44953	26.20225	15.41142	43.63178	1.35709
21	43.84110	35.70912	3.52009	28.88025	12.35624
22	17.06132	18.89226	18.41273	10.81809	30.88893

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 21. Mean forecast error for each linear model at 50% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	13.61592	7.60054	3.69432	8.90872	3.03120
2	17.60957	2.36482	1.06917	13.04887	2.24079
3	0.33347	30.05267	53.30968	8.45276	117.61150
4	16.29746	2.32110	1.31353	8.73170	7.07968
5	1.12782	2.75900	36.08551	4.89571	27.88363
6	3.49774	3.77193	2.44344	2.43223	2.41860
7	42.82140	46.31455	1.36637	19.70370	1.10889
8	1.63417	6.99904	13.48748	3.15276	26.54109
9	37.30332	27.05637	6.42723	28.01674	9.74404
10	4.34496	15.68148	55.99302	15.41195	57.93760
11	1.60842	24.49739	15.02853	3.18163	26.42469
12	2.56089	41.65305	33.51027	7.05876	85.23250
13	27.59739	33.00248	2.30578	15.42340	2.85812
14	65.00580	49.02258	10.58361	48.90004	22.21993
15	27.89112	5.49615	0.40386	16.01284	4.05306
16	13.75667	10.17014	64.36843	29.45980	82.41737
17	6.16326	18.75175	31.38334	2.74513	68.72715
18	14.03759	14.92091	1.86180	10.54053	3.46811
19	31.83621	31.17855	3.26999	12.33937	0.93734
20	30.11817	15.85810	6.14864	23.56138	0.63973
21	30.45439	17.87203	4.20594	23.87537	6.66659
22	23.77345	23.02034	3.07105	17.51270	5.03543

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 22. Mean forecast error for each linear model at 75% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	1.13790	0.80729	4.02012	0.90320	3.95065
2	4.69080	1.38658	0.94592	4.71722	2.02368
3	0.12374	0.43076	18.54060	1.51986	38.06407
4	7.26222	1.42117	0.83406	6.24747	0.93907
5	1.27369	2.61792	19.56769	2.37943	13.02981
6	1.11203	1.63357	0.14627	1.26809	0.14817
7	10.28820	8.41429	1.23882	5.63897	1.17635
8	1.06142	1.66600	0.87894	1.28285	2.08821
9	15.89398	12.24788	3.61722	14.59497	2.83391
10	1.69319	1.86531	21.15078	3.84380	17.71334
11	3.17135	14.58572	16.05304	5.71446	28.91974
12	4.70326	20.94185	26.41417	7.78753	54.69814
13	20.54496	23.37257	1.56817	16.42611	2.26665
14	26.66647	20.15688	4.27299	23.15578	2.56824
15	12.65341	0.77902	0.69399	9.23059	0.28967
16	12.00221	29.40741	45.04551	20.60569	56.20283
17	2.59728	16.35027	24.48050	3.25725	37.77993
18	3.95637	5.16053	0.81626	3.79140	1.09521
19	17.96830	20.23772	0.38999	10.78042	0.83170
20	9.86710	8.64223	2.69552	9.79091	0.41673
21	5.16540	2.64236	0.54451	5.03112	0.71604
22	18.84289	20.48720	4.49913	18.60089	5.03543

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 23. Mean forecast error for each nonlinear model at 5% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	33.346	149.659	12.226	10.195	282.130	190.651
2	134.815	24.301	35.495	97.825	149.914	8.140
3	12.832	272.717	138.047	65.562	298.991	205.190
4	97.287	12.952	7.956	56.369	3.727	62.512
5	1.835	9.632	113.068	27.931	104.871	6.757
6	6.029	521.688	83.151	24.740	531.135	79.877
7	87.990	77.204	2.226	32.007	1.398	45.505
8	1.496	416.611	45.691	5.558	424.653	248.596
9	66.349	52.760	3.890	36.287	4.685	61.690
10	6.026	160.033	65.792	7.746	210.864	120.510
11	30.672	139.440	84.048	61.595	144.921	22.741
12	99.298	3.426	7.663	33.980	76.768	6.163
13	15.244	12.847	40.370	369.155	14.615	7.289
14	574.798	13.250	8.125	24.861	62.650	35.515
15	29.790	215.572	118.317	78.863	218.687	184.609
16	3.448	102.894	61.148	17.823	123.767	43.090
17	214.944	211.124	65.764	161.371	93.569	214.406
18	41.464	24.273	1.078	19.726	4.860	7.798
19	25.660	280.930	194.108	108.576	307.328	116.698
20	82.921	30.395	22.182	59.814	1.162	63.983
21	63.431	49.436	3.233	27.211	4.269	30.401
22	22.543	210.656	24.797	8.377	250.107	45.922

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 24. Mean forecast error for each nonlinear model at 10% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	14.335	141.054	28.193	4.876	263.286	44.063
2	17.704	124.994	8.661	6.828	186.974	132.484
3	8.003	164.091	109.126	42.119	220.983	34.379
4	104.278	64.473	17.986	69.832	28.995	92.394
5	9.254	13.250	25.631	1.142	6.933	1.533
6	3.116	25.638	17.682	2.539	84.632	28.781
7	122.330	137.321	19.675	83.084	53.121	70.124
8	31.182	391.378	116.079	58.721	404.201	225.245
9	64.054	66.245	12.034	44.710	22.920	62.342
10	6.253	123.326	98.326	34.444	159.027	18.448
11	212.517	200.302	66.393	164.146	62.005	144.604
12	74.979	5.993	9.316	40.931	43.701	21.289
13	10.818	11.665	39.845	9.792	31.744	8.079
14	720.277	54.855	7.704	47.989	21.658	68.571
15	111.549	104.924	17.956	78.175	24.040	87.691
16	19.614	12.535	30.191	4.572	60.773	16.429
17	30.927	22.534	19.873	6.205	90.750	6.148
18	32.305	34.951	4.024	21.385	10.492	16.904
19	24.343	17.974	14.505	7.844	4.248	24.742
20	82.921	30.395	22.182	59.814	1.162	63.983
21	57.404	44.927	0.303	33.276	2.462	46.570
22	11.368	63.817	39.749	13.219	97.350	17.507

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 25. Mean forecast error for each nonlinear model at 25% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	10.433	5.605	11.270	5.542	18.692	4.294
2	41.275	4.140	3.498	28.644	2.871	40.128
3	8.073	55.178	78.719	27.054	101.415	13.223
4	2.915	89.552	29.729	6.466	134.781	63.333
5	2.154	13.250	15.348	1.560	3.961	1.272
6	13.531	193.139	59.523	23.808	242.830	215.056
7	53.824	66.873	4.430	36.235	15.351	51.332
8	4.436	32.450	38.313	10.366	87.219	3.603
9	35.820	35.592	8.723	29.414	13.272	31.964
10	12.455	37.289	87.371	33.038	111.750	18.448
11	5.374	34.926	9.411	1.669	52.799	17.359
12	20.806	141.150	71.028	37.407	166.243	153.364
13	20.064	25.039	5.759	11.641	4.068	19.671
14	63.567	63.839	14.321	50.892	24.284	60.475
15	37.434	12.442	3.987	27.367	1.277	3.368
16	26.966	78.303	86.571	49.902	97.884	47.813
17	4.811	66.568	52.234	14.501	114.211	29.343
18	23.387	27.080	7.066	19.651	54.107	18.449
19	26.681	40.222	2.664	21.414	12.058	26.330
20	32.645	16.222	9.507	27.780	1.139	22.370
21	28.521	26.270	3.452	22.503	9.904	17.277
22	18.551	24.414	7.905	13.134	7.575	18.769

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 26. Mean forecast error for each nonlinear model at 50% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	5.150	3.923	3.426	4.974	2.951	3.381
2	4.997	4.140	8.661	5.388	2.794	40.128
3	0.635	1.904	8.395	0.359	4.440	0.630
4	7.127	3.736	0.857	6.309	1.085	5.217
5	2.490	6.730	1.584	2.398	1.194	1.464
6	7.895	11.817	0.851	7.009	3.429	4.169
7	12.900	66.873	1.564	10.429	2.523	9.733
8	1.181	1.987	6.315	1.717	5.772	1.239
9	17.430	15.716	5.144	17.313	6.785	14.011
10	1.380	0.353	22.391	3.099	11.667	2.235
11	4.427	20.730	16.770	6.563	25.137	13.483
12	8.406	28.256	33.662	11.740	56.598	16.153
13	20.360	30.827	4.919	20.260	11.451	19.542
14	32.418	31.807	11.958	32.803	18.394	27.117
15	1.613	2.408	1.397	1.353	7.822	3.086
16	12.149	5.840	41.770	17.718	37.202	11.999
17	9.161	24.188	38.655	13.359	52.354	14.785
18	5.235	5.217	0.965	5.283	1.236	2.566
19	10.510	18.537	0.476	10.326	4.176	9.924
20	11.934	3.845	2.609	11.815	1.956	4.605
21	19.132	15.233	4.274	18.543	6.493	13.449
22	26.700	34.717	8.685	26.268	17.229	26.905

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 27. Mean forecast error for each nonlinear model at 75% estimation level (22 multi-industry cases)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	1.138	1.645	3.688	0.862	4.067	2.332
2	1.613	1.298	1.278	2.446	2.670	1.240
3	0.128	1.439	2.155	0.264	0.622	0.120
4	3.630	3.055	0.823	4.602	1.216	2.595
5	1.341	3.917	1.617	1.701	1.056	1.330
6	1.090	1.451	0.204	1.432	0.304	0.693
7	1.163	1.268	1.254	1.368	0.909	0.680
8	1.841	3.487	0.205	2.375	0.568	0.030
9	7.694	5.821	2.729	9.140	1.855	5.163
10	0.427	0.244	5.570	0.211	1.352	1.199
11	6.108	10.219	16.565	7.128	19.818	8.417
12	7.895	10.157	24.124	8.360	27.036	9.368
13	12.581	16.388	4.536	14.250	7.006	11.025
14	10.006	5.564	3.563	11.425	1.901	5.285
15	2.589	0.379	0.318	2.748	0.535	0.551
16	7.814	4.633	24.356	8.946	16.848	10.161
17	6.147	8.538	21.566	7.200	20.216	7.485
18	1.941	2.202	0.588	2.459	0.517	1.036
19	5.108	9.730	0.490	6.448	2.583	4.701
20	5.707	3.845	2.296	7.006	0.396	2.689
21	3.038	0.926	0.717	3.644	0.618	0.758
22	15.571	17.173	7.603	17.744	9.158	12.631

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 28. Mean forecast error for each linear model at 5% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	181.929	36.777	13.341	96.671	35.147
2	23.278	89.815	15.399	0.662	100.191
3	300.563	34.767	27.602	169.518	7.209
4	81.184	98.348	14.784	7.661	72.235
5	174.186	134.608	2.343	51.303	1.728
6	69.317	32.517	24.986	4.503	25.357
7	77.581	62.304	26.979	3.120	92.179
8	53.230	39.026	13.463	11.374	13.939
9	92.058	50.623	2.411	31.245	93.197
10	205.026	8.482	3.842	74.821	38.844

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 29. Mean forecast error for each linear model at 10% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	129.064	1.672	6.185	60.146	15.137
2	20.460	6.460	10.740	0.831	37.019
3	296.445	126.062	33.037	172.461	48.675
4	83.312	7.521	8.433	12.461	28.924
5	151.361	152.865	2.773	43.419	2.436
6	65.132	31.201	21.186	5.334	22.507
7	79.512	5.017	3.415	7.687	27.499
8	66.786	51.436	6.559	20.399	6.135
9	66.149	13.071	2.741	22.964	8.111
10	170.014	9.575	1.840	53.908	38.724

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 30. Mean forecast error for each linear model at 25% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	37.892	4.883	2.123	9.864	20.320
2	25.703	0.780	3.481	3.713	7.917
3	214.117	63.496	20.215	108.486	4.868
4	76.862	39.777	1.285	21.738	2.026
5	58.011	2.162	0.266	5.647	12.213
6	44.258	36.516	15.731	5.207	15.810
7	73.096	27.754	6.535	11.320	7.897
8	34.996	26.312	8.806	12.354	11.609
9	10.387	4.778	11.470	1.005	26.692
10	44.363	15.272	3.121	4.681	29.467

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 31. Mean forecast error for each linear model at 50% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG
CASE					
1	5.281	9.192	5.888	0.437	13.636
2	10.630	2.926	1.736	2.423	3.638
3	51.207	3.456	0.615	16.011	0.413
4	37.820	8.250	1.539	10.599	2.257
5	18.703	0.066	14.082	0.122	13.693
6	32.222	23.432	11.747	5.537	9.813
7	57.996	33.117	0.164	17.629	0.202
8	34.252	33.259	3.566	17.464	3.933
9	3.954	1.988	9.482	0.078	13.420
10	29.124	3.661	0.021	7.605	0.001

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal

Table 32. Mean forecast error for each nonlinear model at 5% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	152.579	33.735	8.766	76.396	35.550	0.604
2	8.169	90.147	21.159	0.078	99.473	81.475
3	210.048	29.661	15.366	108.439	3.308	144.083
4	70.384	9.247	6.161	2.865	30.451	63.628
5	198.941	207.221	18.143	107.987	43.066	164.069
6	16.574	6.519	33.657	3.265	33.530	5.354
7	21.341	57.131	38.169	7.412	85.006	4.892
8	82.324	82.092	2.066	32.125	3.280	44.551
9	92.058	50.623	2.411	31.245	93.197	16.535
10	39.986	32.364	3.531	3.523	46.349	29.305

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 33. Mean forecast error for each nonlinear model at 10% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	69.911	3.767	2.345	28.770	16.970	0.697
2	15.155	2.972	8.877	0.856	27.531	4.220
3	226.069	139.000	35.561	142.339	52.132	202.462
4	76.159	24.609	0.516	26.959	0.887	70.851
5	51.039	54.715	3.562	12.042	3.446	25.819
6	28.229	22.448	14.346	3.159	9.867	23.879
7	57.373	34.419	1.498	16.113	1.138	52.403
8	70.173	71.233	1.932	32.008	3.936	11.034
9	70.125	27.905	2.733	29.843	3.773	69.927
10	45.317	16.403	1.640	6.933	33.518	0.494

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 34. Mean forecast error for each nonlinear model at 25% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	2.760	10.747	5.177	0.525	21.112	7.840
2	24.632	8.948	0.646	7.814	0.750	21.226
3	26.647	1.537	2.361	13.453	0.334	3.997
4	29.314	24.517	0.639	13.851	0.809	20.147
5	0.547	11.504	19.613	5.626	23.961	5.897
6	5.428	7.654	11.283	2.216	8.554	3.411
7	44.669	48.926	0.606	20.826	3.487	41.763
8	16.280	16.928	7.069	8.925	6.858	11.034
9	0.700	13.253	16.047	2.548	29.918	14.885
10	1.466	12.226	7.963	2.030	20.673	3.889

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic

Table 35. Mean forecast error for each nonlinear model at 50% estimation level (10 telephone company cases of electronic for electromechanical switching)

MODEL	FP	WB	GZ	NM	LG	EX
CASE						
1	0.519	6.895	7.382	1.497	11.547	3.384
2	1.877	0.858	1.096	0.839	1.275	0.446
3	0.143	0.919	0.468	0.004	2.117	0.412
4	4.116	1.703	1.136	1.985	0.840	1.872
5	1.212	4.430	14.299	4.080	12.640	2.670
6	5.685	8.565	4.722	3.297	2.668	4.632
7	15.327	16.508	1.926	11.392	4.161	12.880
8	22.511	31.440	3.430	18.824	6.724	20.637
9	1.199	1.549	8.030	1.131	8.242	1.466
10	6.480	2.314	0.597	4.069	1.046	4.545

Key: FP = Fisher/Pry, WB = Weibull, GZ = Gompertz
NM = Normal, LG = Lognormal, EX = Extended logistic